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Electro-elastic fields around two arbitrarily-shaped holes in a finite electrostrictive solid

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ABSTRACT

Stress and electric field concentration around two arbitrarily-shaped holes in a finite electrostrictive solid subjected to uniform remote electric field are studied. The edge of the solid and the shapes of the holes are defined via several conformal mappings. The electro-elastic field of the solid as well as the electric fields both outside the solid and inside the holes are obtained on using conformal mapping techniques, Faber series and Fourier expansion method. Extensive numerical results are shown for elliptical, triangular, square, oval and rectangular holes in a square electrostrictive plane. The exact external electric field near the edge of the solid is found to be essentially *non-uniform* and its maximum appears to be much larger than the uniform remote electric field, while it is shown that the hoop stress nearby the point of maximum curvature on the hole's boundary increases rapidly as the point of maximum curvature approaches either the edge of the solid or the other hole. On the other hand, for a single centrally-located hole, the finite electrostrictive plane can be approximately treated as an infinite plane if the size of the finite plane is not less than six times the size of the hole, while for two interacting holes, the interaction between them is negligible when the distance between the two holes is larger than three times the size of the holes.

1. Introduction

Electrostrictive materials have been widely used in smart structures and devices due to their high sensibility and quick response. However, for such materials with defects like holes or cracks, a large electric load could induce high electrostrictive stress concentration around the defects which may lead to failure of the materials [1–3]. Therefore, the study of the electrostrictive stress distribution near the defects within electrostrictive materials is of great significance.

In fact, the theory of electrostriction is nonlinear and has been studied by many researchers. Stratton [4] and Landau and Lifshitz [5] obtained the electrostrictive stress induced by electric field in electrostrictive media, while Kuang [6] also discussed the electrostrictive force in detail. Yang [7] derived approximate equations for the extensional and flexural motion of electroelastic plates subject to large electric fields. McMeeking and Landis [8] investigated the polarizable dielectrics under large deformation by considering the electrostatic forces and stored energy. McMeeking et al. [9] developed a principle of virtual work for combined electrostatics and mechanics. Kuang [10] derived some nonlinear variational principles for elastic dielectric materials with electric Gibbs free energy and the complete governing equations. Suo et al. [11] presented a new formulation of nonlinear field theory of dielectrics for finite deformation. Bustamante et al. [12] introduced two new variational principles for

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electroelastostatics based on the electrostatic scalar potential and vector potential combined with the deformation function. Skatulla et al. [13] adopted a strain gradient approach based on a generalized continuum framework to describe the electromechanically coupled behavior of electro-active polymers including scale effects. More recently, Richards and Odegard [14] developed a constitutive model for electrostrictive polymers within a thermodynamic and hyper-elastic framework, and Hong [15] constructed a continuum field theory for a viscoelastic dielectric under combined electric and mechanical loads. Wang and Willatzen [16] developed a nonlinear dynamical model for multi-layer piezo-ceramic transducer systems, while Barakati and Zhupanska [17] studied the dynamic response of electrically conductive composites under a pulsed electromagnetic field by solving the nonlinear equations of motion and Maxwell's equations simultaneously.

In order to obtain some approximate solutions for electrostrictive materials with holes or inclusions, Knops [18] and Smith and Warren [19] developed a complex variable method by adopting a simplified linear dielectric response. McMeeking [20] employed this method to investigate the electrostrictive stresses near a crack-like flaw with relation to the flaw aspect ratio. Ru et al. [21] obtained electric-field induced stress intensity factors of an interfacial crack in multilayer electrostrictive actuators based on the small-scale saturation solutions. Sladek [22] calculated stress intensity and electric intensity factors for cracks in continuously nonhomogeneous piezoelectric body under a transient dynamic load, Loboda et al. [23] obtained the lengths of electrical saturation and mechanical yielding zones near the crack tips inside a thin interlayer bonded to two piezoelectric half planes, and Zhong [24] gave a magneto-electro-elastic solution for two dielectric cracks in a piezoelectric and piezomagnetic solid by solving singular integral equations. Beom [25] and Beom et al. [26] did two-dimensional asymptotic analyses of a semi-infinite crack and an impermeable crack in an electrostrictive ceramic subjected to electric loading under small-scale conditions, respectively. Jiang and Kuang [27–29] presented a series of solutions to the two-dimensional problems of an electrostrictive medium with an elliptical rigid conductor, a crack and an elliptical inclusion, respectively, based on the revision of the formulations of Knops [18] and Smith and Warren [19] by considering Maxwell stresses induced by the applied electric field. Recently, Gao et al. [30,31] gave the two-dimensional solution of a permeable elliptical hole or collinear permeable cracks within an electrostricitve solid under uniform remote electric field, in which the external electric field imposed on the edge of the solid is simply assumed to be identical to the uniform remote one. Actually, the presence of solid perturbs the external electric field so that the external electric field near the edge of the solid is generally not equal to the uniform remote one and thus the Maxwell traction induced on the edge of the solid should be recalculated based on the exact electric field near the edge of the solid. However, such an important effect seems to be ignored in the existing literature. On the other hand, almost all of previous works on the two-dimensional hole/inclusion problems of an electrostrictive material have been limited to a single elliptical hole/inclusion. In fact, the non-elliptical shape of the holes/inclusions as well as the interaction between the holes/inclusions and the edge of the material will have significant impact on the local electric field and stress distribution around the holes/inclusions. Therefore, we are motivated to study the problem of a finite electrostrictive solid containing two holes of arbitrary shape based on the accurate electric field near the edge of the solid, in which the interaction between the two holes and the edge of the solid is taken into account. It is worth noting that the nonlinear electromechanical coupling relation for an electrostrictive material studied here is quite different from the linear one for a general piezoelectric material, so our present problem is absolutely different from that of two arbitrarily-shaped holes in a piezoelectric plane studied in [32] in most significant aspects from study objective to main results achieved.

The paper is organized as follows. Basic equations and a detailed problem description are presented in Sections 2 and 3, respectively. General solutions for the electro-elastic field in the electrostrictive solid and the electric fields both outside the solid and inside the holes are given in Section 4. In Section 5, numerical examples are shown to discuss the effect of the distance between the holes and the edge of the solid on the stress concentration around the holes. Finally, the main results are summarized in Section 6.

2. Basic formulations

In a Cartesian coordinate system (x_1, x_2, x_3), consider an isotropic electrostrictive material without piezoelectricity and free charge density inside the body, and denote the components of stress, strain, displacement, electric displacement and electric field by σ_{kl}, e_{kl}, u_l, D_l and E_l $(k, l = 1\ldots3)$, respectively. The equilibrium equations for the material in electroelastostatics can be expressed as:

$$\frac{\partial D_l}{\partial x_l} = 0, \tag{1}$$

$$\frac{\partial \tilde{\sigma}_{kl}}{\partial x_l} = 0, \tilde{\sigma}_{kl} = \sigma_{kl} + \sigma_{kl}^M, \tag{2}$$

where the repeated indices stand for summation, and $\tilde{\sigma}_{kl}$ and σ_{kl}^M are called the pseudo stress and the Maxwell stress which can be expressed by the electric field components as:

$$\sigma_{kl}^M = \frac{1}{2}\varepsilon(2E_kE_l - E_iE_i\delta_{kl}), \tag{3}$$