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Differential quadrature procedure for in-plane vibration analysis of variable thickness circular arches traversed by a moving point load

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ABSTRACT

Point discretization methods such as the differential quadrature method (DQM) are well known to have difficulties in solving partial differential equations that involve the Dirac-delta function because the Dirac-delta function is a generalized singularity function and it cannot be discretized directly using the DQM. To overcome this difficulty, a simple differential quadrature methodology is proposed in this study, where the Dirac-delta function is expanded into a Fourier trigonometric series. By expanding the Dirac-delta function into a Fourier trigonometric series, this singular function is treated as non-singular functions, which can be discretized easily and directly using the DQM. The applicability of the proposed method is demonstrated by the in-plane vibration analysis of variable thickness circular arches traversed by a moving point load. The numerical results show that the proposed method is highly accurate and reliable.

1. Introduction

The problem of beam-like structures traversed by a moving point load has been an interesting research subject for a long time because this problem affects many applications, such as highway and railway bridges, piping systems subjected to fluid flow, beams subjected to pressure waves, overhead cranes, cables transporting humans/materials, and many modern machining operations [1]. Therefore, it is very important to accurately predict the dynamical behavior and vibration characteristics of these types of structures.

Due to their great practical engineering importance, the dynamics and vibrations of circular arches have been investigated by many researchers in the past. However, most of these studies have focused mainly on the *free vibration analysis* of circular and/or non-circular arches. In particular, Ball [2] proposed a finite difference method for studying the dynamic behavior of rings. Laura and Verniere De Irassar [3] proposed a Ritz approach to study the free vibration of symmetric circular arches with linearly varying thickness and carrying concentrated masses. Auciello and De Rosa [4] presented a comprehensive but brief review of various methods for analyzing the free vibration of circular arches. Chidamparam and Leissa [5] proposed a Galerkin approach to study the effects of the extensibility (or compressibility) of the centerline on the in-plane free vibration of loaded circular arches. Huang et al. [6] proposed a systematic and accurate procedure that incorporates the Laplace transform to calculate the transient response of circular arches. In another study, Huang et al. [7] developed an analytic solution for the in-plane vibration of arches with variable curvature as well as cross- section using the famous

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Frobenius method combined with the dynamic stiffness method. Tüfekçi and Arpaci [8] proposed an exact solution for the free in-plane vibrations of circular arches with uniform cross-section by considering axial extension, shear deformation, and rotatory inertia effects. Tong et al. [9] derived a closed-form solution for the in-plane vibration of inextensible circular arches with a uniform cross-section. The derived exact solution was applied to circular arches with stepped cross-sections and then employed to obtain approximate solutions for arches with non-uniform cross-sections. Lin and Lee [10] derived a closed-form solution for the dynamic analysis of extensional circular Timoshenko beams with general elastic boundary conditions. Kang et al. [11] proposed a wave propagation approach to study the free vibration of planar curved beams. Tüfekçi and Ozdemirci [12] determined the natural frequencies of stepped circular arches both analytically and experimentally. Shafiee et al. [13] obtained closed-form solutions for the mechanical buckling of curved beams with doubly symmetric cross-section when subjected to a uniform distributed radial load and pure bending moment. Shin et al. [14] proposed a differential transformation method for investigating the vibration of inextensible circular arches with variable cross-section. Shahba et al. [15] proposed a modified Adomian decomposition method to analyze the free vibration of non-uniform cross-section, inextensible, circular arches and rings. Wu et al. [16] proposed an exact solution for the free in-plane vibration analysis of circular arches carrying various concentrated elements. Rajasekaran [17] proposed a new differential transformation-based arch element to analyze the static stability and free vibration of arches. Petyt and Fleischer [18], Sabir and Ashwell [19], Raveendranath et al. [20], Litewka and Rakowski [21], Yang et al. [22], Wu and Chiang [23–26], and Yang et al. [27] also proposed various finite element methods to investigate the dynamic behavior of arches.

The differential quadrature method (DQM) has also been used by several researchers to study the dynamic behavior of circular arches, although most of these studies focused on the *free vibration analysis* of *inextensible* circular arches. To the best of our knowledge, the first attempt to apply the DQM to arch problems was described by Gutierrez and Laura [28], who employed the DQM together with the δ-technique to obtain the fundamental frequencies of inextensible non-uniform cross-section rings. Kang et al. [29,30] also used the same procedure to study the dynamic behavior of both extensible and inextensible circular arches, where they obtained the fundamental frequencies of uniform cross-section circular arches with simply supported and clamped boundary conditions. In another study, they investigated the influence of shear deformation on the natural vibration of circular arches [31]. De Rosa and Franciosi [32] proposed a modified version of the DQM to solve the sixth-order differential equation of motion that governs the free in-plane vibration of inextensible circular arches, where the higher-order derivatives at the boundary points were viewed as additional independent variables. Liu and Wu [33] proposed a generalized differential quadrature rule (GDQR) to study the free vibration of inextensible circular arches. In their approach, multiple boundary conditions were applied by assigning two degrees of freedom (displacement and slope) to each boundary point. They also employed a specific set of trial functions (i.e., the Hermite interpolation shape functions) to determine the weighting coefficients of the GDQR. Karami and Malekzadeh [34] proposed a DQM for the in-plane free vibration analysis of circular arches with varying cross-sections. Their method also considered the second-order derivatives of the displacement at the boundary points as the degrees of freedom for the problem. Malekzadeh and Karami [35], Chen [36], Shin et al. [14], Malekzadeh and Setoodeh [37], and Ni et al. [38] also solved various arch problems using the DQM.

Recently, the DQM has been used by some researchers to study the dynamic behavior of circular arches traversed by a moving point load. Nikkhoo and Kananipour [39] proposed a formulation based on the DQM for the transient dynamic analysis of *uniform cross-section inextensible* circular arches subjected to a moving point load, although no details were given of how the time-dependent Dirac-delta function was approximated or treated mathematically in their study. Their numerical solutions also failed to achieve good agreement with the analytical and finite element solutions [39]. As discussed in previous studies [40,41], the main source of error in their numerical results may have been due to incorrect mathematical modeling of the Dirac-delta function.

When the DQM is applied to a problem, the governing differential equation of the problem, including the force function, must first be satisfied at a number of grid points. Clearly, when the force function is a singular function, it cannot be discretized directly using the DQM. To overcome this drawback, this author recently proposed two different approaches. The first approach, which combines the DQM with the integral quadrature method (IQM), was applied successfully to the vibration (or bending) problem for *straight beams* subjected to a moving (or static) point load [41,42]. However, this approach requires the support of another method (i.e., the IQM) to handle the Dirac-delta function. In general, it is more desirable to solve this type of problem by using the DQM itself, rather than by combining it with other methods or techniques. In the second approach, the Dirac-delta function is approximated by a standard Gaussian function, and thus the singular Dirac-delta function can then be treated as a non-singular continuous function, which can be discretized easily and directly using the DQM. This approach has been applied successfully to the vibration problem for *straight beams* and rectangular plates subjected to a moving point load [40]. Recently, Tornabene et al. [43] also employed this technique to solve the bending problem for doubly-curved composite deep shells subjected to a static point load; however, the Gaussian function involves a *regularization parameter*, which needs to be adjusted carefully before the problem can be solved. This parameter was also demonstrated to be a *problem-dependent* parameter [40].

In the present study, to avoid the limitations mentioned above, an alternative approach is proposed where the Dirac-delta function is expanded into a Fourier trigonometric series. By expanding the Dirac-delta function into a Fourier trigonometric series, the singular Dirac-delta function may be treated as a non-singular continuous function, which can be discretized directly using the DQM. Compared with the coupled DQM–IQM approach, the proposed approach does not require the support of any other approach to treat the Dirac-delta function. In addition, compared with the Gaussian function approach, the proposed approach does not involve any *regularization parameter*. The applicability of the proposed method is demonstrated by