Applied Mathematical Modelling 40 (2016) 4664–4680

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A mixed decomposition-spline approach for the numerical solution of a class of singular boundary value problems

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ARTICLE INFO

Article history:
Received 19 August 2012
Revised 17 November 2015
Accepted 24 November 2015
Available online 11 December 2015

Keywords:
Adaptive cubic B-spline collocation method
Modified decomposition method
Singular boundary value problems
Rate of convergence
Patching technique

ABSTRACT

In this paper, we formulate a patching scheme to solve numerically a class of boundary value problems. The strategy is being founded on a merger of two methods, a modified decomposition technique applied on a small interval near the singularity and a fourth order spline collocation technique which is used on the rest of the domain of the problem. The convergence of the proposed method is analyzed and a fourth-order rate of convergence is proved. Some examples are given to test the efficiency of the scheme and to verify the order of the rate of convergence. The numerical results are compared with exact solutions and the outcomes of other existing numerical methods.

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1. Introduction

The objective of this paper is to present a patching strategy for obtaining a numerical solution to the subsequent singular boundary-value problem:

$$(x^{\alpha}y')' = f(x, y), \tag{1.1}$$

which is complimented with the boundary conditions:

$$y(0) = A, \quad y(1) = B. \tag{1.2}$$

Here $0 < \alpha < 1$, $0 < x < 1$, and A, B are finite constants. We will extend the cases for $1 < \alpha \leq 2$ and when the boundary condition $y(b)$ is specified at some point $b > 1$ and for initial condition of the form $y'(0)$ as well. Further, for the existence of a unique solution we require the following assumptions on the function $f(x, y)$ (see Keller [1]):

(a) $f(x, y)$ is continuous,
(b) $\frac{\partial f}{\partial y}$ exists and is continuous,
(c) $\frac{\partial f}{\partial y} \geq 0$ and $|\frac{\partial f}{\partial y}| \leq K$ for some positive constant K.

In recent years, there has been considerable literature on singular boundary-value problems that has been the focal point of a number of authors. Due to the numerical difficulties of handling singularities, an extensive number of articles appeared in which a variety of numerical schemes were introduced and proposed in an attempt aimed at tackling and overcoming singularities. Singular boundary value problems arise commonly in physiology, in chemical and mechanical engineering, physics and

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http://dx.doi.org/10.1016/j.apm.2015.11.045
S0307-904X(15)00795-7/© 2015 Elsevier Inc. All rights reserved.

numerous other applications (see [2,3]). Substantial effort has been made in developing numerical techniques for the solution of singular BVPs. For instance, Jamet [4] has discussed the existence of a unique solution of a class of second order linear singular BVPs and implemented a proposed finite difference approach to obtain a numerical solution of such type of BVPs. Wazwaz [5–7] used a modified decomposition method for solving singular initial value problems, including the Lane–Emden type equations as well as the treatment of the Emden–Fowler equation. Weinmüller [8] utilized collocation methods for solving linear BVPs with a singularity of the first kind. Cohen and Jones [9] investigated a shifted Chebyshev polynomial with the finite deferred correction approach for the numerical solution of a second order linear ordinary differential equation with a regular singular point. Cen [10] treated a class of singular two-point BVPs using a numerical strategy over a non-uniform mesh. Green's function of the singular boundary value problem is used to derive an equivalent integral equation which is then solved by interpolation with quadratic polynomials. Kadalbajoo and Aggarwal [11] presented a numerical solution for singular two-point boundary value problems for certain ordinary differential equation having singular coefficients: to remove the singularity, they first used Chebyshev economizition in the vicinity of the singular point, then the resulting regular BVP is treated efficiently by employing the B-spline collocation technique. Kumar [12] employed a novel three-point finite difference method defined on a uniform mesh for a class of singular two-point BVPs. Kanth and Bhattacharya [13] studied a class of singular BVPs via utilizing a quasilinearization technique that converts the nonlinear BVP to a set of linear differential equations that are then solved numerically by a modified spline collocation approach. Similarly, Kanth [14] used a cubic spline polynomial to solve a class of singular two-point BVPs, in which again the quasilinearization strategy was employed in order to convert the nonlinear problem to a set of linear differential equations, that are modified at the singular point, and then solved numerically by cubic spline. Pandey and Singh [2] proposed a second-order finite difference scheme on a uniform mesh for solving a class of singular BVPs that model problems that appear in physiology. Gustaffsson [15] presented a numerical method for solving singular boundary value problems. Kanth and Reddy [16] treated singular two point BVPs using a Chebyshev economizition technique. Kanth and Reddy [17] proposed a numerical method for solving a two point BVP, defined on [0, 1], that possesses a regular singularity at $x = 0$. Chawla and Katti [18] constructed three-point finite difference approximations for the solution of a class of singular two-point BVPs. Danish et al. [3] presented the optimal homotopy analysis method for the solution of singular boundary value problems. Other established methods are also available, such as the well-known robust piecewise polynomial and adaptive collocation methods that are utilized in recognized codes such as COLSYS (see the paper by Ascher, Christiansen and Russell [19] and the references therein).

In a recent paper, the authors in [20] introduced a strategy based on merging a modified decomposition method and the cubic B-spline collocation for the approximate solution of singular boundary-value problems appearing in physiology. In this paper, a similar but a considerably more improved patching strategy is used for solving the class of singular second-order boundary-value problems given (1.1) and (1.2). The principle idea of this method is to decompose the domain into two subintervals. In the first subinterval, that contains the singularity, a modified decomposition scheme based on a new special type of integral operator is applied to overcome and efficiently handle the singularity. Then in the second subinterval, which does not contain the singularity, the resulting problem is treated via applying an adaptive spline collocation technique [21,22] over a uniform or non-uniform mesh. The efficiency and performance of the approach is examined using a number of numerical test examples. The numerical outcomes indicate that the method is very accurate when compared to existing analytical and/or numerical results. Further, it is verified theoretically and numerically that this technique has fourth-order convergence.

In the past years, there has been an enormous interest (see [23–28]) in the decomposition method for solving nonlinear equations [29]. In this paper, we employ a modified version of the decomposition method in order to efficiently surmount the numerical difficulties arising from the singularity, yet the shortcoming of this method is that it diverges rather rapidly as the applicable domain increases, that is, it converges only locally. In contrast, the spline approach gives a global approximation regardless of the size of the interval, nevertheless it has a downside in tackling the singularity. To avoid the deficiencies of both methods we propose a mixed patching approach that balances and manipulates the advantages of both techniques.

The paper is organized as follows. In Section 2, we describe the patching strategy that merges the modified decomposition and spline collocation schemes and apply it to solve a class of singular second-order BVPs. Convergence results are included and it is shown both theoretically and numerically that the convergence of the method is of order four. In Section 3, a number of test numerical examples are studied to assess the accuracy of the technique. Finally, Section 4 includes a conclusion that briefly summarizes the numerical results.

2. Numerical method

Our method is based on a fourth order non-uniform cubic B-spline collocation, the modified decomposition, and the mixed patching technique formulation. It consists of several elements, which are now described in the next five sections and finally combined into the desired strategy. Discussion of convergence of the method that includes a proof of the fourth-order convergence is presented.

2.1. Non-uniform fourth order spline collocation

In this section, the non-uniform fourth order spline collocation scheme [21] based on non-uniform nodes is described and implemented for the numerical solution of the singular BVP (1.1) and (1.2). Nonetheless, this approach is applicable solely for linear problems and our BVP is nonlinear, therefore to overcome this difficulty an iteration scheme arising from Newton's method is employed in order to linearize the problem.

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