Applied Mathematical Modelling 40 (2016) 4700–4711

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Numerical analysis of dual-phase-lag heat transfer for a moving finite medium subjected to laser heat source

Haw-Long Lee [a], Wen-Lih Chen [a], Win-Jin Chang [a], Ming-I Char [b], Yu-Ching Yang [a,*]

[a] Clean Energy Center, Department of Mechanical Engineering, Kun Shan University, No. 195, Kun-Da Rd., Yung-Kang Dist., Tainan 710-03, Taiwan, ROC

[b] Department of Mechanical Engineering, National Chung Hsing University, No. 250, Kuo-Kuang Rd., Taichung 402, Taiwan, ROC

ARTICLE INFO

Article history:
Received 26 May 2014
Revised 3 September 2015
Accepted 2 December 2015
Available online 12 December 2015

Keywords:
Non-Fourier heat transfer
Dual-phase-lag
Moving finite medium
Laser heat source

ABSTRACT

To accommodate the micro-structural effect, this work applies the dual-phase-lag (DPL) heat transfer model to explore the transient heat transfer for a moving finite medium under the effect of a time-dependent laser heat source. Laser heating is modeled as an internal heat source. A numerical scheme has been developed to overcome the mathematical difficulties in dealing with the hyperbolic heat conduction equation. Comparison between present numerical results and the analytic solutions for the non-Fourier case is made to verify the accuracy of the present numerical method. Additionally, the effects of different medium parameters, for example, moving velocity, phase lags values of the heat flux and temperature gradient, on the behavior of heat transfer have been examined. It is found that there exists clear phase shifts in the temperature distributions due to the medium moving velocity. The heat-flux phase lag tends to induce thermal waves with sharp wave-fronts in the medium, the inclusion of temperature-gradient phase lag smoothens the sharp wave-fronts by promoting conduction into the medium, resulting in non-Fourier diffusion-like conduction.

1. Introduction

Due to its numerous applications related to microelectronics and material processing, the use of laser and microwave heat sources has attracted much attention in recent years. These can be used in processes such as laser patterning, micromachining, and laser surface treatment. In terms of theoretical analysis on heat conduction problems, classical Fourier's law has been widely used; and its predictive accuracy is supported extensively and successfully by results which are in agreement with experimental data for most of the conventional heat transfer conditions. However, in situations involving high heat fluxes, very short times, or very low temperatures, the applicability of Fourier's law is questionable [1,2]. The problem is rooted in the fact that the Fourier's law predicts an infinite speed of heat propagation, that is, a thermal disturbance in any part of a medium can result in an instantaneous perturbation to everywhere in the sample.

To account for the finite thermal wave speeds, Cattaneo [3] and Vernotte [4] proposed a thermal wave model with a single-phase time lag in which the temperature gradient established after a certain elapsed time:

$$q(r, t + \tau_q) = -k\nabla T(r, t), \tag{1}$$

* Corresponding author. Tel.: +88662050780.
E-mail address: ycyang@mail.ksu.edu.tw (Y.-C. Yang).

http://dx.doi.org/10.1016/j.apm.2015.12.005

Nomenclature

A_1	parameter defined in Eq. (27)
A_2	parameter defined in Eq. (28)
A_3	parameter defined in Eq. (25)
B_1	parameter defined in Eq. (30)
B_2	parameter defined in Eq. (31)
B_3	parameter defined in Eq. (29)
c	specific heat ($\mathrm{Jkg^{-1}K^{-1}}$)
d_1, d_2	parameters defined in Eq. (25)
g	capacity of internal heat source ($\mathrm{Wm^{-3}}$)
G	dimensionless capacity of internal heat source
I	laser incident intensity ($\mathrm{Wm^{-2}}$)
I_r	arbitrary reference laser intensity ($\mathrm{Wm^{-2}}$)
k	thermal conductivity ($\mathrm{Wm^{-1}K^{-1}}$)
l	thickness of the slab (m)
L	dimensionless thickness of the slab ($= cl/2\alpha$)
q	heat flux ($\mathrm{Wm^{-2}}$)
r	position vector (m)
R	surface reflectance
$R_1 \sim R_3$	parameters defined in Eq. (20)
s	Laplace transform parameter
T	temperature (K)
T_0	initial temperature (K)
t	time (s)
u	step function
v	speed of heat propagation ($\mathrm{ms^{-1}}$)
w	velocity of the medium ($\mathrm{ms^{-1}}$)
W	dimensionless velocity of the medium
x	x-coordinate (m)

Greek symbols

α	thermal diffusivity ($\mathrm{m^2s^{-1}}$)
β	dimensionless absorption coefficient
$\delta(\xi)$	Dirac δ function
η	dimensionless Cartesian coordinates
θ	dimensionless temperature
μ	absorption coefficient
ξ	dimensionless time
ρ	density $\mathrm{kgm^{-3}}$
τ_q	phase lag of the heat flux (s)
τ_T	phase lag of the temperature gradient (s)
ψ_0	internal heat source
ϕ	dimensionless rate of energy absorbed in the medium

where τ_q denotes the relaxation time or the time delay of the heat flux relative to the temperature gradient in the transient energy transport process. From Eq. (1), it can be seen that when $\tau_q > 0$, the thermal wave propagates through the medium with a finite speed of $v = \sqrt{\alpha/\tau_q}$, where α is the thermal diffusivity. However, when τ_q approaches zero, the thermal wave has an infinite speed and thus the single-phase-lag (SPL) or non-Fourier model reduces to the traditional Fourier model.

Although the thermal wave model can capture the micro-scale response in time [5,6], the wave concept does not capture the micro-scale response in space [7]. Therefore, the validity of the thermal wave model becomes debatable in the aspect of fast-transient response with micro-structural interaction effects [8]. To remove the precedence assumption implied by the thermal wave model, the dual-phase-lag heat conduction model was developed and experimentally verified by Tzou [9]. The model accounts for spatial and temporal effects in both macro- and micro-scale heat transfer in a one-temperature formulation and takes the form:

$$q(r, t + \tau_q) = -k\nabla T(r, t + \tau_T), \tag{2}$$

where τ_T is the phase lag of the temperature gradient. In the DPL model with $\tau_q > \tau_T$, temperature gradient within the medium induces heat flux; hence, the temperature gradient is the cause for energy transport and the heat flux is the effect; whereas, with