Applied Mathematical Modelling 40 (2016) 4727–4745

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An improved multidimensional parallelepiped non-probabilistic model for structural uncertainty analysis

B.Y. Ni, C. Jiang*, X. Han

State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, College of Mechanical and Vehicle Engineering, Hunan University, Changsha City 410082, PR China

ARTICLE INFO

Article history:
Received 18 April 2015
Revised 23 November 2015
Accepted 26 November 2015
Available online 12 December 2015

Keywords:
Non-probabilistic uncertainty quantification
Convex model
Uncertainty propagation
Reliability analysis

ABSTRACT

The non-probabilistic convex model utilizes a convex set to quantify the uncertainty domain of uncertain parameters. Different with "interval model" and "ellipsoid model", the parallelepiped convex model can include the dependent and independent interval variables in a unified framework to deal with the complex "multi-source uncertainty" problems. Based on the existing multidimensional parallelepiped (MP) model, this paper proposed an improved MP model for uncertainty quantification. The correlation coefficient between interval variables in this improved MP model is redefined and an explicit expression describing the uncertainty domain of the interval variables is derived based on the correlation matrix. Through matrix transformation, the parallelepiped-shaped uncertainty domain can be projected into a box. The improved MP model is then applied to the uncertainty propagation analysis and reliability analysis of structures. Several numerical examples are investigated to demonstrate the effectiveness of this model.

1. Introduction

In recent decades, the non-probabilistic convex model theory has received increasing attention by researchers, since to some extent it can be applied to quantify uncertainty when less sample data and information are available. In the early 1990s, the non-probabilistic convex model theory was proposed by Ben-Haim and Elishakoff [1–3], and they advocated the use of convex sets to describe the uncertainty domain of uncertain-but-bounded parameters, which are also called as interval variables, when sample information is inadequate. During the past 20 years, the theory of convex model has been developed in a variety of areas, and a series of achievements have arisen in this field. A comparison of probability model and non-probabilistic convex model was made in Ref. [4]. And in [5], an "uncertain triangle" was employed to describe the relationship between the three kinds of uncertainty quantification models, i.e., probabilistic model, fuzzy sets and convex model. It has also been illustrated that the probability and non-probabilistic convexity concepts are not antagonistic [6]. In our previous work [7], a correlation analysis technique was proposed for uncertain modeling of multidimensional ellipsoid model, which provides a strict mathematical means for construction of the ellipsoidal uncertainty domain. Based on the non-probabilistic convex models and interval analysis method, a comparison was made for dynamic response measures of an infinitely long beam that subjected to a moving force with constant speed [8], and an inverse method was presented to identify the bounds of uncertain dynamic load acting on the structures [9]. With regard to reliability, based on the expansion of convex models, a non-probabilistic measure of reliability for linear systems was proposed [10]. Meanwhile, a possible measure for non-probabilistic reliability was provided [11], in which the structural

* Corresponding author. Tel.: +86 731 88823325; fax: +86 731 8821445.
E-mail address: jiangc@hnu.edu.cn (C. Jiang).

http://dx.doi.org/10.1016/j.apm.2015.11.047
S0307-904X(15)00799-4/

reliability was treated as an interval rather than a deterministic value. The traditional first-order reliability method (FORM) was then introduced into the convex models and two reliability indices were defined for the interval and ellipsoid models [12,13]. Based on the ellipsoidal convex model, a new method was proposed to compute the structural response under load uncertainty [14]. The non-probabilistic convex model is also commonly applied in the field of design optimization [15–18] and in finite element analysis [19–21]. Besides, several reliability-based optimization design methods were developed for structures with convex model uncertainty [22,23]. Other applications of the convex model in engineering mechanics include stability analysis of elastic bars on uncertain foundations [24], bound analysis of structural responses of beams [25], uncertainty analysis in structural number determination in flexible pavement design [26], etc. By combining the probabilistic model with the non-probabilistic convex model, a series of hybrid uncertain models and corresponding reliability analysis methods were formulated [27–31].

Presently, two kinds of convex models are widely used, i.e., interval model and ellipsoid model. With the premise that the variation range of a single variable belongs to an interval, the interval model describes the uncertainty domain of these variables as a multidimensional cuboid; in this case, these interval variables are assumed to be independent with each other, and they are called "independent interval variables" in non-probabilistic convex model. While in ellipsoid model, the uncertain variables are assumed to be mutually dependent and the uncertainty domain is described by a multidimensional ellipsoid. However in practical applications, there exists another typical kind of "multi-source uncertainty" problems [32,33], in which the uncertainties arise from different sources such as material property, external loads, etc. The uncertain parameters from the same source sometimes are dependent while the ones from different sources are mutually independent. The widely used interval and ellipsoid models will in general lead to overlarge uncertainty domain and conservative results if they are employed to deal with the multi-source uncertainty problems. In multi-ellipsoid convex model [32,33], all of the uncertain-but-bounded parameters are divided into uncorrelated groups and each group consists of only dependent uncertain parameters described by a sub-dimensional ellipsoid convex set. In our recent work [34,35], a new kind of convex model, namely, "multidimensional parallelepiped model (MP model)" was proposed, which to a certain extend was a more general convex model and could take into account of independent and dependent interval variables in a unified framework. The MP model uses the marginal intervals of all the interval variables and the correlation coefficients of each pair of interval variables defined in the MP model to construct the uncertainty domain. Theoretically, the MP model could provide a promising way for multi-source uncertainty quantification and corresponding non-probabilstic reliability analysis; however, in current state it still has some deficiencies. Firstly, the existing MP model cannot provide an explicit expression for the uncertainty domain like the quadratic function in ellipsoid model, which makes it inconvenient to use when dealing with some relatively complex engineering problems; secondly, an affine transformation must be employed for structural uncertainty analysis in the MP model, which puts forward a little high mathematical requirement for engineers and hence sets up obstacles for practical applications of the model.

Aimed at the above problems, in this paper we proposed an improved non-probabilistic MP convex model, in which the uncertainty domain of the interval variables can be given an analytical expression in the form of matrix inequality. Subsequently, only a simple matrix transformation is required for subsequent structural uncertainty analysis rather than the affine coordinate transformation. In addition, the uncertainty domain that depicted by the improved MP model is also more reasonable than the previous one. Therefore, the complex multi-source uncertainty analysis can be treated much more conveniently; hence we can substantially promote the practicability of the MP convex model. The remainders of this paper are organized as follows: Section 2 formulates the improved MP model; Sections 3 and 4 apply this improved MP model to uncertainty propagation analysis and reliability analysis, respectively; Section 5 provides four numerical examples and Section 6 gives the conclusions.

2. The improved MP convex model

Uncertainty quantification by non-probabilistic convex model means that the uncertainty domain of the uncertain-but-bounded parameters is characterized through a bounded convex set; and all possible samples of these parameters should be located within this domain theoretically. The convex model is not a single mathematical model; instead, it stands for a series of models. The "interval model" and the "ellipsoid model" (Fig. 1(a)) are the most widely used convex models, which could respectively handle uncertainty problems where independent or dependent interval variables exist. The MP model proposed in our recent work [34,35] can deal with problems where independent and dependent interval variables coexist. And the joint uncertainty domain of the interval variables is depicted by a multidimensional parallelepiped as shown in Fig. 1(b); when the variables are not independent with each other, the MP model degenerates into the interval model.

As mentioned previously, the existing MP model [34,35] has two main deficiencies. First, its uncertainty domain cannot be represented through an explicit expression; second, a relatively complex affine coordinate transformation is involved when applying it to structural uncertainty analysis. In the following text, we will then propose an improved MP model to overcome the above two problems and whereby greatly improve the practicability of the MP model. For easy understanding, we will first create the model for two-dimensional problems and then deal with the general model for multidimensional problems.

2.1. Two-dimensional model

For a two-dimensional problem with interval variables X and Y, the MP model to be built degenerates into a parallelogram, as shown in Fig. 2. Assume that the variation interval that X belongs to is $X^I=[X^C-X^W, X^C+X^W]$, and that of Y is $Y^I=[Y^C-Y^W, Y^C+Y^W]$. The closed intervals X^I and Y^I are called the marginal intervals of interval variables X and Y, where X^C and Y^C represent the midpoints of X^I and Y^I, X^W and Y^W represent their radii, respectively. If interval variables X and Y are