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Interaction between collapsing bubble and viscoelastic solid: Numerical modelling and simulation

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ABSTRACT

In this paper, a fluid–solid interaction algorithm is developed to study the interaction between a collapsing bubble and a deformable viscoelastic solid, which is accomplished by coupled boundary-element (BE) and finite-element (FE) calculations incorporating a Zener type viscoelastic model. The boundary-element method (BEM) is used to simulate the physical process of bubble growth, contraction and collapse while the finite element method (FEM) is used to calculate the viscoelastic solid response to the impulsive pressure induced by the bubble collapse. The implementation of the Zener type viscoelastic model is critical for this study to accommodate the stiffness variation of a viscoelastic solid with time under the pressure. The study reveals the effects of the viscoelastic properties, different maximum bubble sizes and bubble locations on the dynamic response of the viscoelastic solid. Interesting insights into the complex problem of the interaction between a collapsing bubble and a deformable viscoelastic solid are obtained, which can be useful for potential application in biomedicine or marine industry.

1. Introduction

The interaction between an oscillating bubble and a deformable structure is commonly found in marine industrial applications and medical treatments [1]. In view of the need to minimize the destructive effects of bubble pressure on the deformable structure in marine design or to utilize it in medical and clinical therapies, a profound understanding of the interaction between a bubble and a structure with different material properties becomes a requirement. Many researchers have put their efforts concerning this problem on two aspects: the first is how the bubble is influenced by the deformed structure and the second is how the structure is responding to the bubble oscillations.

Bubble dynamics influenced by different boundaries have been studied extensively. Gibson and Blake [2,3] performed some of the earliest theoretical and experimental studies on the interaction between an oscillating bubble and an elastic boundary. They found that the flexibility of the boundary is crucial to the response of a nearby cavitation bubble. Shima et al. [4] conducted a set of experiments with spark-generated bubbles in the vicinity of a compliant surface and observed different migratory behaviour during bubble collapse. Laser-induced cavitation bubbles near a flexible membrane [5], composite surface [6], elastic boundary [7,8], flat rigid surface and flat free surface [9] were also studied. Turangan et al. [10] studied a non-equilibrium bubble placed next to a (stretched) membrane. Ohl et al. [11] conducted low-voltage spark bubble experiments and adopted a boundary-element method (BEM) model to predict the dynamics of oscillating bubbles placed near a thick layer of elastic biomaterial. Recently,

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bubble dynamics near a perforated plate in a vertical cylinder was simulated using a combined boundary element-finite difference method [12], and evolution features of a toroidal bubble were investigated using an improved 3D bubble dynamics model based on BEM [13].

On the other hand, how a deformable structure responds to the impulsive pressure induced by an oscillating bubble has also been investigated. Duncan et al. [14,15] have explored the structural response of a compliant boundary to bubble oscillation. In their studies, the compliant surface has been modelled as a membrane with a spring foundation [14], or using a finite element representation [15]. Both reports found good qualitative agreement with the experiments reported by Shima et al. [4]. Chahine's group [16,17] developed the three dimensional bubble dynamics code 3DYNAFS and coupled it to FEM structural dynamics codes. They used their codes to simulate the interaction between a free-floating surface piercing body that interacts with and responds to a simulated underwater explosion bubble. Klaseboer et al. [18] studied the underwater explosion bubble dynamics and its interaction with simple flat plates numerically and experimentally using a finite-element method (FEM) and boundary-element method (BEM) code. Gong et al. [19] studied the physical behaviour of the interaction of a spark-generated bubble and an elastic rubber beam numerically and experimentally. In their study, the scaling law for spark-generated bubbles and underwater explosion bubbles of Gong et al. [20] was used to get the parameters for the spark-generated bubble.

All the preceding studies listed above, however, deal with a deformable (elastic) structure response to pressure induced by an oscillating bubble. The modelling and simulation for the interaction of a collapsing bubble with a deformable viscoelastic structure appears very scarce. In fact, the great majority of biological materials are viscoelastic to a greater or lesser extent [21], polymers are viscoelastic at all temperatures [22]. Therefore, it is necessary to extend our investigation into the interaction of a collapsing bubble with a deformable viscoelastic solid. The present work addresses this and proposes a feasible modelling and simulation method for this complex problem. In this study, the boundary-element method (BEM) is used to simulate the fluid surrounding the deformable viscoelastic solid, including the bubble growth, contraction and collapse. The BEM is particularly appealing to model moving interfaces; since only one layer of mesh is needed on the boundaries of the fluid tied to the deformable viscoelastic solid and the bubble surface. The finite element method (FEM) is used to calculate the viscoelastic solid response to the impulsive pressure induced by the bubble motion. The interaction of the collapsing bubble and the viscoelastic solid is simulated using a coupled BEM and FEM code with a Zener type viscoelastic model.

2. Coupled fluid–solid interaction algorithm

2.1. Basic equations for the fluid

The fundamental assumption is that the flow in the time-varying fluid domain is inviscid, incompressible and irrotational. This implies that there exists a potential function Φ in the fluid domain which may be bounded by bubbles and moving solid structures. The velocity vector obeys: $\mathbf{v} = \nabla\Phi$. This equation, together with the continuity equation $\nabla \bullet \mathbf{v} = 0$, leads to the Laplace equation which is valid anywhere in the fluid:

$$\nabla^2\Phi = 0. \tag{1}$$

The Laplace equation is an elliptic equation, this means that the solution can be computed everywhere in the fluid domain, if either Φ (Dirichlet condition) or $\partial\Phi/\partial n$ (Neumann condition) is given on the boundaries of the problem. The z-axis is pointing upwards as shown in Fig. 1. Green's second identity can be used with the Green's function $G(\mathbf{p},\mathbf{q}) = 1/|\mathbf{p}-\mathbf{q}|$as:

$$c(\mathbf{p})\,\Phi(\mathbf{p}) + \int_S \Phi(\mathbf{q})\frac{\partial G(\mathbf{q},\mathbf{p})}{\partial n}\,dS = \int_S G(\mathbf{q},\mathbf{p})\frac{\partial \Phi(\mathbf{q})}{\partial n}dS, \tag{2}$$

where $\boldsymbol{p}$ and $\boldsymbol{q}$ are vectors pointing to a fixed and an integration point, respectively. Both points are situated on the boundary S; $\mathbf{c}(\boldsymbol{p})$ is equal to 4π when $\boldsymbol{p}$ is located in the fluid domain, it is 0 when $\boldsymbol{p}$ is located outside the fluid domain and $c(\boldsymbol{p})$ equals the solid angle when $\boldsymbol{p}$ is located on the boundary surface S. Let V_0 and p_0 denote the initial volume and gas pressure at the time of inception of the bubble, the gas pressure inside the bubble is supposed to be spatially uniform and is updated adiabatically from the ratio of the volume: $p_b = p_0(V_0/V)^\gamma$. The time-dependent Bernoulli equation can be applied on the bubble surface:

$$\rho\frac{\partial\Phi}{\partial t} = p_{ref} - p_v - p_0\left(\frac{V_0}{V}\right)^\gamma - \frac{1}{2}\rho|\mathbf{v}|^2 - \rho g z, \tag{3}$$

and on the wet surface of a solid:

$$\rho\frac{\partial\Phi}{\partial t} = p_{ref} - \frac{1}{2}\rho|\mathbf{v}|^2 - \rho g z - p_s, \tag{4}$$

where ρ is the density of the fluid, g is the gravity acceleration, p_{ref} and p_v are the reference pressure and the vapour pressure, and p_s is the pressure of the fluid in contact with the wet surface of the solid .

Numerically, Eqs. (3) and (4) can be solved with Eq. (2) for Φ if the initial conditions which initiate the bubble are known. However, the initial conditions in practice are not always available. Gong at al. [20] have proposed to use a small equivalent explosive charge weight W to determine the initial conditions of a bubble according to a scaling law for different bubbles. The maximum radius of a bubble induced by an underwater explosion in terms of the charge weight W can be estimated by [20]:

$$R_m = \left(\frac{3K_{EW}}{4\pi\rho g}\right)^{1/3} \times \frac{W^{1/3}}{\left(D + \frac{p_{ref}-p_v}{\rho g}\right)^{1/3}}. \tag{5}$$