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A novel analytical-numerical solution for nonlinear time-dependent electro-thermo-mechanical creep behavior of rotating disk made of piezoelectric polymer

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ABSTRACT

A new analytical-numerical method has been developed for electro-thermo-mechanical transient creep response of rotating disk made of nonlinear piezoelectric polymer. The viscoelastic properties are stress, temperature and time dependent. The disk has been placed in an axisymmetric distributed temperature field and is subjected to a centrifugal body force with free-free, free-fixed and fixed-free boundary conditions. The Burgers' viscoelastic creep model for long time prediction has been employed. Using the potential-displacement equation, charge equation of electrostatics, strain-displacement equations, stress-strain relations and equilibrium equation, the constitutive differential equation in terms of displacement is obtained. The non-homogeneous part of which contains creep strains. An analytical solution for radial displacement in terms of creep strains has been performed. Then the stresses, strains and electric potentials are also derived in terms of creep-strains. Prandtl-Reuss relations and the material creep model are employed to obtain history of displacement, stresses, strains and electric potential. It is concluded that the displacement is increasing with time while effective stresses are decreasing. A significant redistribution occurs for the electric potential which is due to stress redistribution during the lifetime of the disk. The results are validated with ABAQUS software. The analytical-numerical method can be used to predict the history of displacement and stresses in all structures made of nonlinear viscoelastic material.

1. Introduction

The application of piezoelectric polymers in smart structures is quite considerable and the analysis of such structures has been an active area of research in the past decade. The mechanical deformation of piezoelectric materials produces electric potential while imposing electric potential difference on the piezoelectric materials produce mechanical deformation. Piezoelectric materials are extensively used to make sensors and actuators.

Polymeric elements are at risk of creep deformation even at low room temperatures. However, these components are extensively used at higher temperatures. When polymeric piezoelectric components are subjected to thermomechanical loads the electric potential can also be applied, the combination of which will produce a complicated thermo-electro-mechanical stresses. These initial stresses will be varied with time due to creep. Piezoelectric polymers are used in sensors or actuators,

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Nomenclature

$u(r)$	Radial displacement (m)
r	Radial variable (m)
r_i, r_o	Inner, outer radii (m)
E	Elastic modulus (N/m^2)
e_{33}, e_{13}	Piezoelectric constants (C/m^2)
$\in_{ii}(i=1,2,3)$	Dielectric constant (C^2/N m^2)
$\sigma_i(i=r,\theta,z)$	Component of stresses (N/m^2)
$\varepsilon_i(i=r,\theta,z)$	Component of strains
$\dot{\varepsilon}_i(i=r,\theta,z)$	Component rate of creep strains (1/s)
$\dot{\varepsilon}_e$	Effective rate of strain (1/s)
σ_e	Effective of stress (N/m^2)
$D_i(i=r,\theta,z)$	Components of electric displacement (C/m^2)
$\phi(r)$	Electric potential (W/A)
$T(r)$	Temperature distribution (K)
$\alpha_i(i=r,\theta,z)$	Coefficients of thermal expansion (1/K)
υ	Poisson's ratio

the long time performance of which can be affected by creep deformation. Transient creep analysis of these components, which gives variation of the stresses, strains and electric potential with time, is important for the remnant life assessment and reliability of these elements. Non-stationary structural creep analysis subjected to thermal, mechanical and electrical loads has been studied by many investigators. Loghman and Wahab [1] investigated the creep damages of thick-walled tubes under thermo-mechanical loading using Mendelson's method of successive approximation and the theta projection concept. Zhou and Kamlah [2] have shown that ferroelectric piezoelectric ceramics exhibit significant primary type creep effects even at room temperature which could be represented by a power law constitutive equation. You et al. [3] have investigated the steady-state creep of thick-walled cylinders made of functionally graded materials subjected to internal pressure.

A general axisymmetric method for creep analysis of primary and secondary creep stages has been presented by Jahed and Bidabadi considering both time-hardening and strain-hardening theories [4]. Hoseini et al. [5] have proposed an analytical solution to obtain the steady state creep in rotating thick-walled cylinders subjected to internal and external pressures. They used Norton's power law of creep to derive general expressions for stresses and strain rates in the rotating thick cylinder.

Huang et al. [6] have investigated the transient and steady-state nanoindentation creep of polymeric materials using the Burgers viscoelastic model and power-law creep deformation for transient and steady state creep, respectively.

Loghman et al. [7] have investigated the transient creep behavior of a thick-walled FGM cylinder subjected to temperature and magnetic fields under internal pressure. A semi-analytical solution followed by method of successive approximation has been presented by Ghorbanpour Arani et al. [8] for time-dependent thermo-electro-mechanical creep behavior of piezoelectric cylinder made of PZT-5. Singh and Gupta [9] investigated the effect of anisotropy on creep stresses and creep rates in the FGM cylinder and compared with an isotropic FGM cylinder. They studied the creep behavior of the FGM by a threshold stress based creep law.

Using a semi-analytical method, Loghman et al. [10] studied the creep response of FGM spheres under internal pressure, distributed temperature and uniform magnetic fields.

Loghman et al. [11] presented a semi-analytical solution for thermo-electro-magneto-mechanical creep behavior of FGPM rotating disk. In this paper only free–free boundary condition was considered and the stress histories were obtained using stress rates and time increments. They concluded that major electric potential redistribution occurs due to significant tensile radial stress redistribution. Thermoelastic creep behavior of FGM rotating disk with variable thickness has been considered by Hosseini Kordkheili and Livani [12]. Histories of electric potential, stresses and radial displacement in a piezoelectric sphere under internal pressure and temperature field have been studied by Ghorbanpour Arani et al. [13]. Initial thermoelastic and time-dependent creep evolution response of rotating ferritic steel disk has been carried out by Daghigh et al. [14] using the "Theta Projection Concept". Loghman and Tourang [15] have considered non-stationary electro-thermo-mechanical creep response and smart deformation control of thick-walled sphere made of polyvinylidene fluoride. In this paper the emphasis is on the smart control of creep deformation by applying suitable electric potential to return back deformations produced during creep process.

Apart from a couple of studies, carried out by a few authors, little or no reference has been made so far in the literature on the time-dependent creep analysis of polymeric piezoelectric rotating disk subjected to electro-thermo-mechanical loading with nonlinear material behavior. In this study however, history of radial displacement stresses, strains, and electric potential of a rotating disk made of radially polarized piezoelectric polymeric material with nonlinear behavior, such as polyvinylidene fluoride (PVDF) is investigated using the new numerical procedure in conjunction with Mendelson's method of successive elastic solution.