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Mathematical model of transmission dynamics of human immune-deficiency virus: A case study for Yunnan, China

Tailei Zhang[a,*], Yicang Zhou[b]

[a] School of Science, Chang'an University, Xi'an 710064, China
[b] Department of Applied Mathematics, Xi'an Jiaotong University, Xi'an 710049, China

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ABSTRACT

In this paper, we present an HIV/AIDS epidemic model with 12 compartments in Yunnan province, China. The total population is divided into four subgroups: injecting drug users(IDUs), female sex workers (FSWs), clients of FSWs (C) and men who have sex with men (MSM). Due to behavioral change, susceptible people will move into the other susceptible groups. A basic reproduction number $\mathcal{R}_0$ of the model is established by means of a next generation matrix. It is found that the model is locally as well as globally asymptotically stable at the disease free equilibrium when $\mathcal{R}_0 < 1$. If $\mathcal{R}_0 > 1$, permanence of HIV/AIDS for the model is studied. Based on HIV/AIDS epidemic data in Yunnan province, parameters are chosen to fit the data. The simulations indicate that Yunnan will have about 140,000 HIV positives, 18,000 AIDS cases unless there are any stronger or more effective control measures by the end of 2015. Sexual transmission is the main mode from 2006–2015. The HIV prevalence rate among men who have sex with men continues to increase more quickly until 2015. If effective measures are taken to reduce the transmission rate, then HIV/AIDS spread in Yunnan can be controlled.

1. Introduction

China's first Acquired Immune Deficiency Syndrome(AIDS) case was identified in 1985 in a dying tourist. In 1989, the first indigenous cases were reported as an outbreak in 146 infected heroin users in Yunnan province, near China's southwest border. The estimation results indicate that by the end of December 2011, the current Human Immune-deficiency Virus (HIV) positives were approximately 780,000(range 620,000–940,000). The proportion of females is 28.6%. The HIV infection rate among China's population is 0.058% (range 0.046–0.070%) [1]. By the end of October 2012 in Yunnan province, the cumulative total of reported HIV positives was 104,981, including 73,397 HIV positive cases, 31,584 AIDS cases and 17,268 recorded deaths [2].

The web reporting data shows great differences in reported cases between different provinces. The top six provinces (Yunnan, Guangxi, Henan, Sichuan, Xinjiang and Guangdong from high to low) of reported HIV positives account for 75.8% of the total national reported cases [1]. The most common factors in which HIV/AIDS is spreading throughout China include needle sharing among intravenous drug users, unsafe sexual activities and sharing contaminated blood products. Yunnan locates in far southwest of China. Because Yunnan borders the 'Golden triangle' which has been one of the most extensive

* Corresponding author. Tel.: +86 13991942722.
E-mail address: t.l.zhang@126.com (T. Zhang).

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Table 1
Definitions of some model parameters.

Parameters	Description
π_1	Transmission probability of HIV per shared needle
π_2	Transmission probability from male to female
π_3	Transmission probability from female to male
π_4	Transmission probability from male to male
c_1	Needle-sharing rate (a number of times per year) of a IDU with other individuals
c_2	Sex-buying rate which is a number of times for a client who buys sex per year
c_3	Sex-selling rate which is a number of times for an FSW who sells sex per year
c_4	A number of sexual contacts of a MSM with individuals per year
$k_i, i = 1, 2, 3, 4$	The proportion of still having high risk in AIDS patients
$d_j, j = 2, 3, 4$	Probability of using condom during one sex activity for jth group
d_1	Probability of not sharing needle for an IDU when he/she need drug

opium-producing areas of Asia and of the world since the 1920s. In 1989, HIV was detected amongst injecting drug users (IDUs) in Yunnan province. Needle sharing drove the epidemic and HIV spread rapidly to IDUs in neighboring cities and along drug trafficking routes. By 2002, HIV was present among IDUs in all China's inland provinces. It is believed that IDUs may have been the core source for all later sub-epidemics in China [3]. By 1999, HIV infections were reported in all 16 prefectures and 111 counties in Yunnan [4]. Yunnan's first HIV infection in MSM was identified in 1999 [4]. Since 2005, the Chinese government has strengthened its intervention efforts to MSM, developed national working protocols and guidelines on HIV prevention and control among MSM, and convened national technical workshops on comprehensive HIV prevention interventions among MSM.

Mathematical models have been used extensively to study the dynamics of HIV/AIDS [5–13]. Nicolas et al. use a mathematical model of the HIV/AIDS epidemic in Kunming, the provincial capital of Yunnan, to study the transmission of HIV/AIDS among IDUs and sex workers [6]. Xiao et al. construct an HIV/AIDS model with 31 patches to understand the epidemic trend in China and to accurately predict the future [12]. Pan and Wu develop a deterministic compartmental model for a population stratified by genders and ages to study HIV spread in China [14]. Vaidya and Wu use an SIM epidemic model to evaluate the role of a seasonal labor-migration to India on HIV transmission in Far-Western Nepal and to assess prevention programs. The joint effects of vaccines and widespread treatment with anti-retroviral therapy (ART) have also been theoretically examined by Blower et al. [7]. They concluded that widespread ART that reduces infectiousness combined with low efficacy vaccine could reduce infections to very low levels as long as behavioral reversals do not overwhelm the reductions in the risk of transmission per sexual act. For example, the Asian Epidemic Model (AEM) has shown that in Dehong prefecture of Yunnan, the proportions of incident HIV infections through sexual transmissions were 50.6%, 52.3% and 52.7% respectively from 2005 to 2007 [15]. In the paper [13], Zhang et al. divided the total population which is restricted within high risk population into two subgroups: injecting drug users (IDUs) and people engaged in commercial sex (PECS) which includes female sex workers (FSWs), and clients of FSWs. Due to this category, the predicting results appears some difference with the actual cases in Yunnan. More importantly, we cannot know the HIV transmission in FSWs or Clients of FSWs. Motivated by aforementioned works, we extend the model in [13] to a more detailed HIV/AIDS model for better understanding the HIV/AIDS prevalence in high risk population. Here, the total population will be divided into four classes: IDUs, FSWs, Clients of FSWs and MSM.

The organization of this paper is as follows: In the next section, we introduce an HIV/AIDS model with 12 compartments. Section 3 is devoted to formulation of basic reproduction number $\mathcal{R}_0$ of the model. In Section 4, we analyze the global stability of the disease-free equilibrium under the condition $\mathcal{R}_0 < 1$. In Section 5, we study the permanence of HIV/AIDS for the model. Numerical simulations are provided in Section 6. We make a short-time prediction for the patterns and prevalence rates of the HIV epidemic in Yunnan province. The simulations also show that the spread of HIV/AIDS may be lowered if the effective actions (condom, Methadone, ART, etc.) are taken to reduce the transmission rates of the high-risk populations.

2. Model formulation

Individuals can go through three states: susceptible (S), HIV-positive but without AIDS (I) and AIDS (A).

The model classifies the high risk population into four groups: (1) injecting drug users(IDUs), (2) female sex workers (FSWs), (3) clients of FSWs(C) and (4) men who have sex with men (MSM). The susceptibles are defined by their behaviors, i.e. if a susceptible individual drugs with others by sharing injectors at time t, then he/she belongs to IDU group $S_1(t)$. The infectives and AIDS are defined by their transmission modes, i.e., $I_1(t)$ stands for all HIV-positive individuals who are infected via injecting drug use at time t. In Table 1 we define the incidence rates of four groups.

Consider one infective client buys sex with a frequency c_2 from FSWs. The probability of not using condom is $1 - d_2$, the probability for the woman to be infected is S_2/N_2 and the probability for transmission of HIV to take place from man to