Applied Mathematical Modelling 40 (2016) 4986–4999

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Reliability analysis of processes with moving cracked material

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ARTICLE INFO

Article history:
Received 23 February 2015
Revised 1 October 2015
Accepted 7 December 2015
Available online 18 December 2015

Keywords:
Moving material
Fracture
Stochastic model
First passage time
Ornstein–Uhlenbeck process

ABSTRACT

The reliability of processes with moving elastic and isotropic material containing initial cracks is considered in terms of fracture. The material is modelled as a moving plate which is simply supported from two of its sides and subjected to homogeneous tension acting in the travelling direction. For tension, two models are studied: (i) tension is constant with respect to time, and (ii) tension varies temporally according to an Ornstein–Uhlenbeck process. Cracks of random length are assumed to occur in the material according to a stochastic counting process. For a general counting process, a representation of the nonfracture probability of the system is obtained that exploits conditional Monte Carlo simulation. Explicit formulae are derived for special cases. To study the reliability of the system with temporally varying tension, a known explicit result for the first passage time of an Ornstein–Uhlenbeck process to a constant boundary is utilised. Numerical examples are provided for printing presses and paper material.

1. Introduction

There are systems in industry in which material moves unsupportedly between two rollers under a longitudinal edge tension. Such systems can be found, e.g., in manufacturing and printing of paper. In paper machines and printing presses, the tension is essential for the transport of the material and it is created by a velocity difference of the rollers. The relative velocity difference of the rollers is called draw, and the span between the rollers is called an open draw.

To achieve good productivity in systems with moving material, there is a demand for running the system at a high speed but at the same time avoiding runnability problems. In pressrooms, runnability problems include web breaks, register errors, wrinkling and the instability of the paper web [1]. Of these problems, especially web breaks have gained attention in the print industry [2].

One of the suspected causes of web breaks in pressrooms are defects. Defects in a paper web can be classified into two categories: microscopic and macroscopic defects. Microscopic defects originate from the natural disorder in paper, such as formation, local fibre orientation and variation of wood species [3]. Macroscopic defects are introduced during the paper-making and transportation processes. In papermaking, a condensation drip in pressing or drying section or a lump on press rolls or press felt can cause holes in the paper web [4]. Such defects occur randomly or in a fixed pattern. Stress formed from running a high roll edge through a nip may cause cracks on the edge of the paper web [4]. Edge cracks of such origin typically occur randomly in the sheet. Insufficient roll edge protection during handling and storage may also cause edge cracks. A cut or nick in the edge of the roll cause multiple edge cracks in the sheet in a localised area [4].

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http://dx.doi.org/10.1016/j.apm.2015.12.010

Web breaks occur at random intervals and they are rare events in pressrooms [5]. Thus, data from a large number of rolls is required for determining the causes of web breaks with a reasonable level of confidence [6] and such data is difficult to obtain under controlled conditions [7]. In addition to the rarity of web breaks, there are often many dependent random variables involved in the printing process, and controlling of them may appear difficult [7]. To avoid these problems, two approaches for finding causes of web breaks have been suggested [7]. One is to conduct data-analysis on massive pressroom data bases and the other is to investigate the effect of different factors on web breaks by mathematical modelling.

Although the effect of macroscopic defects have gained attention in the research (see, e.g., literature review in [2]), to the author's knowledge, only a few studies aim to predict the connection of macroscopic defects and web breaks by mathematical modelling. Swinehart and Broek [8] developed a web break model, based on fracture mechanics, which included the size distribution of flaws, web strength and web tension. In [8], the tension was regarded as constant. Uesaka and Ferahi [7] studied the effect of cracks on web breaks by a break-rate model based on the weakest link theory of fracture. The number of breaks per one roll during a run was derived by considering the strength of characteristic elements of the web. In [7] the tension in the system was assumed to be constant and later, Hristopulos and Uesaka [9] presented a dynamic model of the web transport derived from fundamental physical laws. In conjunction with the weakest link fracture model, the model allows investigating the impact of tension variations on web break rates.

The break-rate model used in [7,9] predicts the upper estimate of the break frequency. However, considering an upper bound of fracture probability may lead to an overconservative upper bound for a safe range of tension. The studies of mechanical instability suggest that the higher the tension, the higher the velocity of the moving material can be [10]. Thus, from the view point of maximal production, an overconservative tension is undesirable as it underestimates the maximal safe velocity.

Motivated by paper industry, defects have also gained attention in the studies of instability of moving materials. Banichuk et al. [11] studied an elastic and isotropic plate that has initial cracks of bounded length travelling in a system of rollers. In [11], the plate was assumed to be subjected to constant or (temporally) cyclic in-plane tension and the Paris' law was used to describe the crack growth induced by tension variations. The optimal average tension was sought for the maximum crack length by considering a productivity function which takes into account both instability and fracture. Moreover, an attempt to take the stochasticity of systems with moving material into account was made in the study by Tirronen et al. [12] in which the safe transition of elastic and isotropic material with initial cracks was analysed by modelling the problem parameters as random variables. In [12], critical regimes for the tension and velocity of the material were sought by considering the probabilities of fracture and instability.

Although tension in a printing press is known to change in time due to draw variations [2] and tension fluctuations have been suggested to cause web breaks [13], the tension in the system was regarded as constant in [7,8]. In [12], the tension was assumed to be constant while a crack travels through an open draw although the constant value was assumed to include uncertainty. In [11], only deterministic variations of tension were considered although the draw variations contain white noise in addition to specific high/low frequency components [2]. In a printing press, cyclic tension variations may be caused by out-of-round unwind rolls or vibrating machine elements such as unwind stands (see [14] and the references therein). In addition to cyclical variations, tension may vary aperiodically due to poorly tuned tension controllers, drives, or unwind brakes ([14] and the references therein). The net effect of such factors cause the tension to fluctuate around the mean value [14].

This study aims at developing mathematical models for systems in which a moving cracked material travels under longitudinal tension. The material is assumed to be elastic and isotropic, and the models of this study focus on describing the occurrence of defects in the material and tension variations in the system, taking into account the stochasticity of these phenomena. This paper extends the study [12] by modelling the crack occurrence and temporal variations of tension by stochastic processes, which enables examination of system longevity. Instead of estimating the fracture probability from above, the present paper aims at directly computing the fracture probability predicted by the model.

Two different models are considered for temporal value of tension. The first model describes tension as constant with respect to time. The second model describes the tension as a stationary Ornstein–Uhlenbeck process. With the latter model, tension has a constant mean value, the set tension, around which it fluctuates temporally. The Ornstein–Uhlenbeck process can be considered as the continuous-time analogue of the discrete-time AR(n) process. It provides a mathematically well-defined continuous-time model for fluctuations of systems whose measurements contain white noise [15, Chapter 4]. Moreover, a stationary process describes random fluctuations of a system which has settled down to a steady state and whose statistical properties do not depend on time [15, Section 3.7]. The stationary Ornstein–Uhlenbeck process can be regarded as a simplified model of tension variations in a printing press.

In this study, we consider straight-line through thickness cracks perpendicular to the travelling direction and located on the edge of the material. Sharp edge cracks oriented in the cross direction of the paper web are most critical in printing presses [16]. Other stochastic quantities in the presented model describe the occurrence of cracks in the open draw and the lengths of the cracks. The locations of the cracks in the travelling direction are described by a stochastic counting process. The lengths of the cracks are modelled by independent and identically distributed (i.i.d.) random variables.

The reliability of the system is studied in terms of fracture by applying linear elastic fracture mechanics (LEFM). For a general counting process, the nonfracture probability is obtained by utilizing conditional Monte Carlo simulation which is one of the most effective techniques for variance reduction [17, Section 5]. An explicit representation is derived for a few special cases. When there is stochastic volatility in tension, considering the probability of a fracture leads to first passage