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Leakage-delay-dependent stability analysis of Markovian jumping linear systems with time-varying delays and nonlinear perturbations

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ABSTRACT

This study is concerned with the problem of leakage-delay-dependent stability analysis of Markovian jumping linear systems with time-varying delays and nonlinear perturbations. Mixed time-varying delays, which include leakage, discrete, and distributed delays, pertaining to the proposed system are addressed. Based on an improved Lyapunov–Krasovskii functional with triple integral terms and by employing the model transformation technique and the reciprocal convex method, the sufficient conditions for the delay-dependent stability of the considered system are derived. Moreover, the sufficient conditions obtained are formulated in terms of linear matrix inequalities to achieve the global asymptotical stability in mean square of the considered delayed system. Finally, numerical examples are provided to demonstrate the less conservatism and effectiveness of the proposed results.

1. Introduction

Time-delay is one of the most significant phenomena that occurs in many different fields such as biology, chemistry, economy and communication networks. Due to the existence of time-delays, dynamical systems may exhibit certain complex, dynamic behaviors, such as oscillation, divergence, chaos, instability, or other poor performance. In this regards, many researchers have conducted in-depth research on stability of time-delay systems in recent years (see e.g., [1–7]). Besides that, stability analysis is of prime importance in the design and application of dynamical systems. Therefore, stability behaviors of dynamical systems have become an important research field in different domains, which include quadruple-tank process [8], controlling two mass-spring system [9], population models [10] and controlling yaw angles of a satellite system [11]. As an example, the quadruple water tank process can be expressed as a delayed nonlinear system. The process consists of four interconnected water tanks and two pumps. The voltages to the pumps are the inputs while the water levels of two lower tanks are the outputs. The main objective is to control the water level of two lower tanks using two pumps, and the stability analysis of the process is important.

In generally, the stability criteria can be classified into two categories, i.e., delay-dependent and delay-independent. The delay-dependent stability criteria are less conservative than delay-independent ones when the size of time-delay is small. Therefore, much attention has been paid for the delay-dependent stability results with mixed time-delays including leakage,

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discrete and distributed delays. As an example, the stability analysis of neural networks with leakage delays has been studied in [11–16]. The problem of stability analysis for dynamical systems with discrete and distributed delays have been given in [17–21]. In particular, a typical time-delay called *leakage* delay may exist in the negative feedback terms of many practical systems. Generally, these terms are known as *forgetting* or *leakage* terms, and they have the tendency to destabilize a system. Leakage delays are encountered in many systems such as population dynamics models [12] and neural networks [13]. The authors of [15] studied the exponential stability of bidirectional associative memory neural networks with time-varying delays in the leakage terms, where the sufficient conditions have been obtained by using fixed point theorem. The effects of leakage delays in a class of nonlinear differential equations have been given in [16]. By using the model transformation technique, the conditions to ensure the existence, uniqueness, and global asymptotic stability of the equilibrium point have been developed. The problem of robust stability analysis for uncertain neutral systems with discrete and distributed delays have been proposed in [20], whereas, global robust asymptotical stability of stochastic neural networks with discrete and distributed time-varying delays have been investigated in [21].

Many dynamical systems have different structures and parameters due to random and abrupt variations, sudden environmental changes, changes in subsystem interconnections. Such type of systems can be modeled as Markovian jump systems, which can be described by a set of linear systems with the transitions between models determined by Markov chain in a finite set. These systems are usually realized in various fields such as failure prone manufacturing systems, power systems, economics systems and aircraft control. Since Krasovskii and Lidskii [22] introduced the concept of Markovian jump systems, a great deal of attentions have been devoted to them in [23–26], and the references therein. For instance, the problem pertaining to the stability analysis of Markovian jump neural networks with mode-dependent delays have been investigated in [24,25], while the problem of H_∞ control and H_∞ filtering for uncertain Markovian jump systems have been considered in [23,27]. Furthermore, He and Liu [28] studied the robust stochastic stability of Markovian jump systems with uncertainties and distributed delays. The global asymptotic stability of stochastic genetic regulatory networks with Markovian jumping parameters have been examined in [29]. In [30], the stability analysis of Markovian jumping neural networks with leakage, discrete and distributed time-varying delays through impulsive control has been considered.

On the other hand, when constructing a dynamical model, uncertainty is unavoidable due to environmental noise, as well as uncertain slowly varying parameters that break up the stability of the system. Therefore, considerable efforts have been paid on the stability of nonlinear perturbed systems with and without time-delays [31–36]. In [31], both delay-independent and delay-dependent stability criteria have been proposed for a time-delayed system under nonlinear perturbation. Delay-dependent stability analysis of neutral systems with mixed time-varying delays and nonlinear perturbations has been dealt by authors in [32,34,36]. In [35], the problem of exponential stability of switched neutral systems with mixed time-varying delays and nonlinear perturbations has been investigated. To the best of our knowledge, research into the stability analysis of Markovian jump systems with nonlinear perturbations, leakage and distributed time-delays is limited. Therefore, it is imperative to improve the stability conditions for Markovian jumping systems with nonlinear perturbations and various time-delays.

Motivated by the above account, a leakage-delay-dependent stability analysis of Markovian jumping linear systems with time-varying delays and nonlinear perturbations is proposed in this paper. Here, it should be noted that various mixed time-delays, which include leakage, discrete, and distributed delays, are treated as time-varying delays. An improved Lyapunov–Krasovskii functional (LKF) with triple integral terms is introduced to derive the sufficient conditions, and is expressed in terms of linear matrix inequalities (LMIs) by using the model transformation technique and the reciprocal convex method. The obtained LMIs can be easily solved using standard numerical software. Finally, numerical examples are presented to illustrate the usefulness and effectiveness of the proposed method.

Notations: Throughout this paper, superscripts T and -1 mean the transpose and the inverse of a matrix respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. For symmetric matrices P and Q, $P > Q$ (respectively, $P \geq Q$) means that matrix $P- Q$ is positive definite (respectively, non-negative). I_n, 0_n and $0_{m,\,n}$ stand for $n \times n$ identity matrix, $n \times n$ and $m \times n$ zero matrices, respectively and the symmetric term in a symmetric matrix is denoted by $\star$, $\varepsilon\{\cdot\}$ represents the expectation operator and $\|\cdot\|$ denotes the Euclidean norm of a vector.

2. Problem description and preliminaries

Given a probability space $(\Omega, \mathbb{F}, \mathbb{P})$ where Ω is the sample space, $\mathbb{F}$ is the algebra of events and $\mathbb{P}$ is the probability measure on $\mathbb{F}$. In this probability space, consider the following system with time-delays and nonlinear perturbations:

$$\begin{aligned}\dot{x}(t) &= A_0(r_t)x(t-\sigma(t)) + A_1(r_t)x(t-\tau(t)) + A_2(r_t)\int_{t-d(t)}^{t} x(s)ds + G(r_t)f_1(r_t, t, x(t-\sigma(t)))\\ &\quad + H(r_t)f_2(r_t, t, x(t-\tau(t))),\\ x(t) &= \phi(t), t \in [\bar{\tau}, 0], \bar{\tau} = \max\{\tau_2, \sigma, d\},\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A_0(r_t)$, $A_1(r_t)$, $A_2(r_t)$, $G(r_t)$, $H(r_t) \in \mathbb{R}^{n\times n}$ are known constant matrices, $\{r_t\}$ is a continuous-time Markovian process with right continuous trajectories taking values in a finite set $S = \{1, 2, \ldots, N\}$ with