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A Viscous Potential Flow model for core-annular flow

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ABSTRACT

Stability of core-annular flow has been studied extensively using spectral and pseudo-spectral methods. These methods, being computationally intensive, render the understanding of the role of each parameter on stability numerically cumbersome. Viscous Potential Flow (VPF) theory offers itself as an alternative analytical tool towards understanding stability of multiphase flows. VPF theory is known to successfully predict the stability features of capillary instability associated with jets flowing in an infinite ambient fluid analytically. In this work, we investigate the applicability of VPF theory in capturing the onset of capillary instability in core-annular flow (CAF). Here, first a VPF stability analysis of core-annular flow is performed for a base state satisfying VPF equations such that each phase has a uniform velocity (plug flow). This potential flow assumption of the base state as well as perturbed variables helps us to obtain a completely analytical dispersion relation to the problem. It is shown that the results of the analysis agree with the published literature of unconfined jets under different limiting conditions. Using VPF also helps to understand the effect of each parameter on the stability features of the system. To improve the accuracy of VPF analysis, we incorporate viscous pressure correction to ensure that continuity of tangential velocity and tangential stresses at the fluid–fluid interface are satisfied in an average sense. We show that the viscous correction to the irrotational pressure results in the same dispersion relation as that obtained by the method of energy dissipation. However, it is shown that this analytical dispersion relation is unable to predict the stability features of CAF even with the inclusion of pressure correction. Next, the stability of CAF is analyzed with laminar core-annular velocity profile as the base state. The results of this VPF analysis are compared with the stability of the complete CAF problem. It is shown that the predictions of the VPF stability analysis for the *cut-off* wavenumber and the *fastest growing* wavenumber associated with capillary instability are in close agreement with that of the complete problem for low Reynolds number and Weber number regimes.

1. Introduction

Core-annular flow (CAF) is a commonly observed flow configuration in confined liquid–liquid and gas–liquid systems under a range of operating conditions [1–3]. The flow is characterized by the presence of one fluid flowing in the central core surrounded by an annulus of the other fluid. An ideal CAF corresponds to the pure co-axial flow of both the fluids with no radial and azimuthal velocity components. This ideal CAF flow configuration has been experimentally observed under

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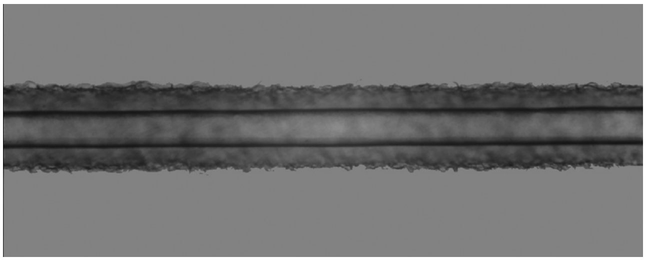

Fig. 1. An ideal core-annular flow (CAF) observed in our experiments carried out in a 150 × 150 μm glass microchannel. The core fluid is toluene and the annular fluid is an aqueous solution of butyric acid.

a narrow range of fluid properties and operating conditions [1]. Fig. 1 shows an ideal core-annular flow observed in our experiments conducted in a micro-channel. Frequently, waves are observed on the interface of ideal CAF and such a flow is called wavy core-annular flow (WCAF). Bamboo waves and flying core are such flow configurations observed in these systems [1]. A theoretical analysis of the ideal CAF helps to analyze the different physical interactions among flow variables and improve our understanding of the origin of flows like WCAF [1]. Excellent reviews on core-annular flow, their stability and applications exist in the literature [1,3,4].

Stability of core-annular flows is of wide interest since it arises in varied applications ranging from lubricated oil transport to process intensification studies involving extraction, reaction and mass transfer in micro-channels. There is a significant amount of literature on the stability of core-annular flows [5–9]. It is known that core-annular flow is rendered unstable by three mechanisms, viz., interfacial tension, interfacial friction and Reynolds stresses [9]. Using energy analysis, Hu and Joseph showed that the interfacial tension mode (corresponding to capillary instability) dominates at low Reynolds numbers [8]. As the Reynolds number is increased, they showed that the core-annular flow becomes stable for a narrow range of parameter space. On further increase of Reynolds number, interfacial friction mode and Reynolds stress modes are observed. Interfacial friction refers to the instability caused by a jump in the velocity gradient across the interface. The third mechanism refers to the onset of Tollmien–Schlichting type of instability driven by inertial forces in the fluids [7,8].

In microfluidics, where the Reynolds and Weber numbers are low (typically Re and $We \sim 100$ or below), the dominant mechanism of instability is due to interfacial tension, also known as capillary instability. This is evident from the experimentally observed mono-disperse drops and slugs in micro-channels. The focus of this work is primarily to analyze systems in this regime. Capillary instability of a jet in a microchannel is frequently exploited to produce uniform sized drops of the core fluid. The size of the drops is determined by several parameters such as viscosity ratio, density ratio, core-to-wall ratio, Reynolds number and Weber number. These parameters interact in a complex manner to determine the final drop size. A linear stability analysis to determine the *fastest growing* wavenumber can help to predict the experimentally obtained drop sizes [7]. In this paper, we propose the application of Viscous Potential Flow (VPF) theory to study the capillary instability of core-annular flows in microchannels. VPF theory helps in mathematically simplifying the analysis, while retaining important physics. This can be used to study the effect of each parameter on the fastest growing wavenumber and determine the drop size. We restrict our analysis to axisymmetric disturbances as they are the most unstable [7,8]. Azimuthally asymmetric perturbations are generally stabilized by surface tension; even if unstable, they tend to have lower growth rates compared to the axisymmetric mode.

In VPF theory, the flow is assumed to be irrotational and the velocity is described by the gradient of a potential. The assumption of irrotationality of the flow eliminates viscous terms from the Navier–Stokes equation. The fluid is however viscid and viscous stresses are non-zero. The viscous stresses are therefore retained in the normal stress balance at the fluid–fluid interface. VPF is known to describe stability of liquid–liquid systems accurately at moderate Reynolds numbers and gas–liquid systems at low Reynolds number [10]. It is expected of VPF theory to work better for gas-liquid interfaces, due to the minor dynamical effects of the gas, than for liquid-liquid interfaces where viscous effects are much larger. VPF provides a computationally elegant method to capture the onset of instabilities for such scenarios. The inclusion of viscous stresses in the normal stress boundary condition is critical in the analysis of interfacial instabilities like capillary instability that are dominated by normal stresses at the interface. In this work, two cases of VPF stability analysis for core-annular flow are discussed. First, in Case I, both the base state and perturbed variables are assumed to satisfy VPF equations. The linear stability analysis based on this approach results in a dispersion relation which is completely analytic.

Once the analytic dispersion relation is obtained, we incorporate a viscous correction to VPF as proposed by Joseph et al. to improve the accuracy of the prediction [10]. In the VPF approach, the continuity of tangential velocity and tangential stress at the interface are not imposed. This limitation is overcome by incorporating a viscous pressure correction to the irrotational solution of the perturbed fluid flow [10]. This viscous correction ensures that continuity of tangential velocity and tangential stress at the interface are satisfied in an average sense. This corrected model is called viscous correction to VPF (VCVPF) and was proposed by Joseph and Wang [11]. This method has been used to study the effect of viscosity on a number of multiphase problems, viz. stability of a liquid jet into gases and liquids [12], capillary instability of a viscous fluid in a gas [13] as well as in another viscous fluid [14], Kelvin–Helmholtz instability with heat and mass transfer in planar [15] as well as cylindrical channels [16].