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Robust H_∞ finite-time stability control of a class of nonlinear systems

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ABSTRACT

In this paper, a robust H_∞ finite-time stability control approach is proposed for a class of nonlinear systems without solving the Hamilton–Jacobi equation and the Riccati equation. First, the concept of robust H_∞ finite-time stability is proposed and the corresponding theorem is presented. Then, for a class of common dynamic systems such as robotic manipulators, a robust H_∞ finite-time stability controller is designed based on the backstepping method, with which the closed-loop system is not only global finite-time stable but also has L_2-gain less than or equal to γ. Finally, some simulations are performed on a robotic manipulator with two degrees of freedom. The results indicate that the proposed control approach is of high effectiveness.

1. Introduction

Like a robotic manipulator, a kind of highly nonlinear, time-varying and coupled complex dynamic system, there always exist uncertainties and external disturbances that may result in control difficulty and performance degradation. In order to achieve higher tracking precision and better transient performance of nonlinear dynamic systems such as high-speed robotic manipulators, continuous finite-time stability control has been paid more and more attentions to by researchers in recent years. Unlike the asymptotic stability, finite-time stability [1] means that system states can be stabilized to equilibrium in finite time, and, it may give rise to fast transient and high-precision performances, which are very useful for high-quality industrial robot control systems. Until now several finite-time control approaches were proposed, such as a finite-time stability approach of homogeneous systems [2,3], a finite-time Lyapunov stability approach [4–7], a terminal sliding mode approach [8–13], and a super-twisting sliding-mode approach [14–18] . Feng et al. proposed a non-singular terminal sliding mode controller for robot systems [11]. Hong et al. firstly proposed a finite-time controller with state feedback and dynamic output feedback control [19], and proposed a non-smooth finite-time stabilization design of nonlinear systems with parametric and dynamic uncertainties by using the input-to-state stability property and the backstepping technique [20]. Su presented a global finite-time tracking controller by replacing linear errors in linear proportional-derivative plus scheme with nonsmooth but continuous exponential errors [7]. Zhao et al. designed a robust finite-time stability controller based on the backstepping method [6]. Su and Zheng implemented the global finite-time tracking of robotic manipulators by replacing linear errors with nonlinear exponential-like errors [21], and addressed the finite-time tracking of robot manipulators in the presence of actuator saturation [22]. However, in all these above-mentioned approaches, Although the robustness of

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nonlinear systems has been considered in some papers, the relationship between the robustness and the control parameters, is not discussed.

Recently, the H_∞ control of nonlinear systems has become a hot point due to its excellent control performance. However, the H_∞ control theory of nonlinear systems, which derives from the L_2-gain analysis based on the concept of energy dissipation, is difficult to apply due to the solvability of the Hamilton–Jacobi equality or inequality. Jongguk and Hyeon transformed robot dynamics into an affine form to achieve a more tractable form by a nonlinear matrix inequality and obtained the approximate solution [23]. Lu et al. investigated the robust H_∞ control of a class of nonlinear systems with parameter uncertainty and determined the sufficient conditions for the existence of the dynamic output feedback controller [24]. Park and Chung proposed a robust linear PID motion controller to solve the nonlinear L_2-gain attenuation control problem of robotic manipulators [25]. Aguilar et al. developed the nonlinear H_∞ controller synthesis for nonsmooth time-varying systems via measurement feedback and Riccati equations [26]. Feng et al. proposed the robust stabilization and robust H_∞ control for uncertain discrete-time singular systems with state delay based on robust stability analysis [27]. And Feng et al. established the sufficient conditions to make singular systems be admissible and strictly (Q, S, R) – α-dissipative, and have H_∞ performance [28]. However, in most conditions, the closed-loop system is asymptotically stable and the H_∞ control parameters have to be obtained by solving the Hamilton–Jacobi equality (or inequality) and the Riccati equation, which results in high complexity for control of robotic manipulators. However, to the authors' best knowledge, there are no results on robust H_∞ finite-time control for a class of uncertain nonlinear systems, which are not only theoretically significant, but also important from an application point of view, such as robotics. This motivates the present study.

Based on the above considerations, in order to guarantee the finite-time convergence and improve the performance of a system with uncertainties and disturbances, the robustness of finite-time stability control is studied and the robust H_∞ control problem is discussed in this paper. In the investigation, firstly, we define the concept of robust H_∞ finite-time stability, and give the relative robust H_∞ finite-time stability theorem. Next, by taking a class of common nonlinear dynamic systems as the research object, a robust H_∞ finite-time control approach without solving the Hamilton–Jacobi equality (or inequality) and the Riccati equation is proposed based on the backstepping method. Then, for complicated and nonlinear robotic manipulators, we design a robust H_∞ finite-time stability controller to obtain finite-time convergence and disturbance attenuation in the L_2-gain sense. Finally, some simulations are carried out on a robotic manipulator with two degrees of freedom to verify the developed approach.

The remainder of this paper is organized as follows: Section 2 briefly reviews the Lyapunov theory for finite-time stability of continuous systems and the relative lemmas; Section 3 gives the main results of this paper, including the definition of robust H_∞ finite-time stability, the robust H_∞ finite-time stability theorem and the corresponding controller; Section 4 discusses a robust H_∞ finite-time stability controller for robotic manipulators; and, in Section 5, some simulations are carried out to verify the proposed control approach.

2. Preliminaries

The concepts related to finite-time stability control are given in Ref. [1]. Here, we recall the corresponding Lyapunov stability theorem for finite-time stability of autonomous systems [29], which will be used in Section 3.

Definition 2.1. Considering a time-invariant nonlinear system in the form of:

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n, \tag{1}$$

where $f : \hat{U}_0 \to R^n$ is continuous in an open neighborhood $\hat{U}_0$ of the origin, the equilibrium $x = 0$ of the system is (locally) finite-time stable if:

(i) It is asymptotically stable in $\hat{U}$, an open neighborhood of the origin, with $\hat{U} \subseteq \hat{U}_0$;
(ii) It is finite-time convergent in $\hat{U}$, that is, for any initial condition $x_0 \in \hat{U}\backslash\{0\}$, there is a settling time $T > 0$ such that every solution $x(t, x_0)$ of system (1) is defined with $x(t, x_0) \in \hat{U}\backslash\{0\}$ for $t \in [0, T)$ and satisfies:

$$\lim_{t\to T} x(t, x_0) = 0. \tag{2}$$

And, if $t \geq T$, $x(t, x_0) = 0$. Moreover, if $\hat{U} = R^n$, the origin $x = 0$ is globally finite-time stable.

Clearly, global asymptotic stability and local finite-time stability imply global finite-time stability. In addition, finite-time stability implies the uniqueness in forward time of the solution $x = 0$.

Definition 2.2. Considering a nonlinear system in the form of:

$$\dot{x} = f(x, u), \quad f(0, 0) = 0, \quad x \in R^n, \ u \in R^m, \tag{3}$$

where f: $R^n \times R^m \to R^n$ is globally defined, the origin of system (3) is finite-time stable if there is a state feedback control law in the form of $u = \phi(x)$ with $\phi(0) = 0$, such that the origin of the closed-loop system $\dot{x} = f(x, \phi(x))$ is in finite-time stable equilibrium.

Lemma 2.3 [2]**.** *Considering nonlinear system* (1)*. Supposing that there is C^1 function $V(x)$ defined in neighborhood $\hat{U} \subset R^n$ of the origin, and real numbers* $c > 0$ *and* $0 < \alpha < 1$, *such that*