# The analysis of a rigid rectangular plate on a transversely isotropic multilayered medium

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## ARTICLE INFO

## ABSTRACT

A semi-analytical method is developed for the analysis of a rigid rectangular plate on a transversely isotropic multilayered medium. Based on the governing equations of elasticity in Cartesian coordinates, transfer matrices for a single layer are deduced through a double Fourier transform and a Laplace transform. The transfer matrix solution of a multilayered medium is subsequently obtained with the consideration of the continuity conditions between two adjacent layers. By the application of the mixed boundary conditions of the contact problem, a pair of 2-D dual integral equations are derived, which are further converted to linear equations by means of the Jacobi orthogonal polynomials. The results of numerical calculation carried out by the corresponding computer program agree well with those obtained from the FEM software ABAQUS. Other numerical examples are presented to elucidate the influence of transversely isotropy, thickness and stratification of the medium.

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## 1. Introduction

The interaction of a rigid plate with an elastic medium is a subject of considerable interest in mechanics. In the researches of contact problems, the primary task is to choose a proper constitutive law of the medium. In most cases, soils in geotechnical engineering are transversely isotropic due to long-term sedimentation processes. Refs. [1–7] provided the fundamental solutions of the transversely isotropic half-space and full-space subjected to different loads. In many situations, soils present the stratification property, so it is more realistic to regard soils as transversely isotropic multilayered media. Numerous methods have been developed to calculate the transversely isotropic multilayered medium, which we may classify as finite layer method [8,9], layer matrices method [10], propagator matrix method [11–14], and analytical layer-element method [15].

In the past, various analytical and numerical methods have been employed to study the response of a loaded plate on an elastic medium. Harding and Sneddon [16] tackled the problem of the frictionless normal indentation in a half-space by a rigid punch with the technique of integral transforms. Collins [17] studied a thin rigid inclusion embedded in an isotropic elastic solid under prescribed displacements. Selvadurai [18] obtained the fundamental result concerning the settlement

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of a rigid circular foundation on an elastic half-space due to an adjacent surface load. Pak and Gobert [19] presented the analytical formulation for the axisymmetric problem of a vertically-loaded rigid disc embedded in a semi-infinite elastic solid.

All studies mentioned above consider neither the transversely isotropy nor the stratification of the medium. On the other hand, Dahan and Zarka [20] investigated the axisymmetric contact problem between a rigid sphere and semi-infinite transversely isotropic body. Fabrikant [21] obtained the exact solution for the contact interaction between a circular punch and a transversely isotropic solid when tangential displacements are specified within the contact area. Yu [22] solved a class of indentation problems in an elastic transversely isotropic half-space by integrating the elastic displacement due to point loads with appropriate distribution functions. By applying double Fourier transforms to deduce a 2-D integral equation, Rahman [23] dealt with an elastic static problem concerning the normal shift of a rigid elliptical disc embedded in a transversely isotropic full-space. Katebi et al. [24] researched the interaction of a vertically loaded disc embedded in a transversely isotropic half-space by deducing a pair of dual integral equations, which can be reduced to a Fredholm equation of the second kind. A review of their studies indicates that the existing solutions mainly concerns about a circle plate resting on or embedded in a transversely isotropic half-space in polar coordinates, while the solutions for a rectangular plate on a transversely isotropic multilayered medium are seldom reported. In fact, certain kinds of substructures such as thick raft foundations and box foundations are generally simplified as rigid rectangular plates in practical engineering. Therefore, the analysis of a rigid rectangular plate on a transversely isotropic multilayered medium in Cartesian coordinates is of great significance in practical engineering.

In this paper, we proposed a semi-analytical method to solve the 2-D dual integral equations dealing with the contact problem of a rigid rectangular plate on a transversely isotropic multilayered medium. The transfer matrices of a single layer in the Fourier transform domain are deduced by means of a double Fourier and Laplace transform. According to the continuity conditions between adjacent layers, the transfer matrix solution of a multilayered medium is further obtained. Considering the mixed boundary conditions, a pair of 2-D dual integral equations of the contact stress is derived and solved. An example of a rigid rectangular plate on a transversely isotropic single-layered medium is performed to confirm the accuracy of the present method. Further numerical examples are given to investigate the influence of the properties of transversely isotropy, thickness and stratification of the medium.

## 2. The transfer matrices of a single layer

It should be noted that the studied problem in this paper is of small deformation. In the Cartesian coordinate system, the equilibrium equations regardless of body force can be expressed as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0, \tag{1a}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0, \tag{1b}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0. \tag{1c}$$

where $\sigma_x$, $\sigma_y$ and $\sigma_z$ represent the normal stress components in the $x$, $y$ and $z$ directions, respectively; $\tau_{yz}$, $\tau_{xz}$ and $\tau_{xy}$ stand for the shear stress components in the planes $yz$, $xz$ and $xy$, respectively.

The constitutive equations of transversely isotropic material, which have five independent elastic parameters, can be written in terms of displacements as follows:

$$\sigma_x = c_{11}\frac{\partial u_x}{\partial x} + c_{12}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z}, \tag{2a}$$

$$\sigma_y = c_{12}\frac{\partial u_x}{\partial x} + c_{11}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z}, \tag{2b}$$

$$\sigma_z = c_{13}\frac{\partial u_x}{\partial x} + c_{13}\frac{\partial u_y}{\partial y} + c_{33}\frac{\partial u_z}{\partial z}, \tag{2c}$$

$$\tau_{yz} = c_{44}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right), \tag{2d}$$

$$\tau_{xz} = c_{44}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right), \tag{2e}$$

$$\tau_{xz} = \frac{c_{11} - c_{12}}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right), \tag{2f}$$