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A fractional sliding mode for finite-time control scheme with application to stabilization of electrostatic and electromechanical transducers

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ABSTRACT

Chaos suppression of non-autonomous uncertain chaotic systems is a very important and attractive topic in the field of physics and engineering. On the other hand, the use of fractional calculus in both research and practice has become as an increasing and interesting issue in recent years. In this paper, we introduce a novel fractional control method for chaos control of integer-order non-autonomous chaotic systems. It is assumed that the system is disturbed by some model uncertainties and external noises. A novel fractional nonsingular terminal sliding manifold which is appropriate for integer-order systems is proposed. Then, on the basis of fractional Lyapunov stability theorem, a suitable robust sliding mode control law is designed to force the state trajectories of the system into the sliding manifold. It is proved that both the sliding mode and reaching phase are realized in a given finite time. Finally, the proposed method is applied for stabilization of chaotic electrostatic and electromechanical transducers.

1. Introduction

In recent years, chaos control has become very attractive and has been applied in vast areas of physics and engineering [1]. Up to now, many control methods, such as sliding mode control [2], optimal control [3], adaptive control [4], linear feedback control [5], nonlinear feedback control [6], backstepping method [7] and H_∞ approach [8], have been successfully applied to control/synchronize the chaos of chaotic systems. However, most of the aforementioned papers have focused on the control of chaotic systems without considering the effects uncertain terms and external noises. While, in practice, some uncertain terms and external noises exist in the chaotic systems' dynamics. Therefore, stabilization of the chaotic systems with model uncertainties and external disturbances is effectively significant in practical applications. In this line, some researchers have proposed a number of techniques, such as nonlinear feedback control [9], sliding mode control [10–12] and adaptive control [13], for chaos control of uncertain chaotic systems.

On the other hand, all of the mentioned above works concentrated on the asymptotic stabilization of chaotic systems. In other words, they have guaranteed that the state trajectories of the chaotic system can converge to the origin over the infinite horizon. While, from a practical standpoint it is more valuable to stabilize the chaotic system in a given finite time. In order to achieve fast convergence in a given time, finite-time control methods have been known to be efficient. Finite-time stability means the optimality in settling time. Furthermore, the finite-time control strategies have demonstrated better

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robustness and disturbance rejection properties. In the works [14–30], we have proposed finite-time controllers for chaos control/synchronization of chaotic systems.

The theory of fractional calculus has a long and prominent history. In fact, one may trace it back to the very origins of differential calculus itself. Such mathematicians as Riemann, Euler and Liouville made some serious research along the lines of non-integer order derivatives and integrals. However, for many years, the theoretical complexity of the fractional calculus prevented it from practical applications. Nevertheless, applications of fractional-order control techniques in chaos control [31–33] and optimization problems [34] have been addressed in the literature. Moreover, fractional calculus has some interesting features such as having a memory of all past events and precise modeling ability of real world systems.

Nowadays, many researchers believe that the fractional modeling of the traditional integer-order systems opens a wide door for research in the area of physical and applied systems, as it truly serves as a generalization of the integer-order case. In other word, the traditional models of many systems need to be revisited within the framework of the fractional-order differential equations, where the integer-order transfer functions of systems become simply special cases of the fractional-order ones.

Thus, the analysis and application of fractional-order controls are by no means trivial. Therefore, the application of non-integer-order controllers for the traditional dynamical systems may obtain more stability and transient behavior characteristics of the dynamics and can suggest more practical and desired control schemes for dynamical systems.

Recently, the application of sliding modes [35] for controlling some systems has attracted the attention of fractional calculus researchers. For example: Tavazoei and Haeri [36] have proposed an active sliding mode approach for synchronization of fractional-order chaotic systems. Hosseinnia et al. [37] have designed a linear sliding surface with corresponding switching law for synchronization of two identical two-dimensional fractional-order chaotic systems. However, in the works [36,37] the stability discussion has not been performed on the basis of the fractional-order Lyapunov theory. But, we have proposed some fractional nonlinear controllers for stabilization/synchronization of fractional-order chaotic systems based on the fractional Lyapunov stability theory [38–45].

In this paper, we discuss the problem of finite-time chaos control of non-autonomous chaotic systems with model uncertainties and external noises. We propose a novel fractional nonsingular terminal sliding mode manifold and prove its finite-time stability using the fractional Lyapunov theorem. Then, a sliding mode control law is designed to ensure that the state trajectories of the chaotic system reach to the proposed sliding manifold in a given finite time. Our main contribution in this paper is the use of a novel non-integer-order nonsingular terminal sliding mode approach for finite-time stabilization of uncertain integer-order non-autonomous dynamical systems. In other to show the effectiveness and usefulness of the proposed scheme, the chaotic behavior of electrostatic and electromechanical transducers is suppressed using the proposed method.

The rest of this paper is organized as follows. In Section 2, some preliminaries of fractional calculus are presented. Chaos control problem formulation is given in Section 3. In Section 4, the design procedure of the proposed fractional-order terminal sliding mode control is provided. Section 5 presents some numerical simulations. Finally, conclusions are given in Section 6.

2. Preliminaries of fractional calculus

Here, the main definitions of the applied fractional derivative/integral are restated.

In the fractional calculus literature, there is a uniform formula for the fractional integral of a function $f(t)$ with $\alpha \in (0,1)$ which is defined as follows.

$$ {}_{t_0}I_t^{\alpha} f(t) = {}_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \tag{1} $$

where $\Gamma(.)$ is the well-known Gamma function.

There are several definitions in the literature for the fractional derivatives. An important and commonly adopted definition of the fractional derivatives of order α of function $f(t)$ is the Riemann–Liouville fractional derivative which is defined below.

$$ {}_{t_0}^{RL}D_t^{\alpha} f(t) = \frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, \tag{2} $$

where $m-1 < \alpha \leqslant m$ and $m \in N$.

Property 1. For the Riemann–Liouville derivative, we have [46]

$$ {}_{t_0}^{RL}D_t^{1-\alpha}\left({}_{t_0}^{RL}D_t^{\alpha} f(t)\right) = {}_{t_0}^{C}D_t^{1} f(t) = \dot{f}(t). \tag{3} $$

Property 2. The following equality is hold for the Riemann–Liouville derivative [47].

$$ {}_{t_0}^{RL}D_t^{\alpha}\left({}_{t_0}^{RL}D_t^{-\alpha} f(t)\right) = f(t). \tag{4} $$