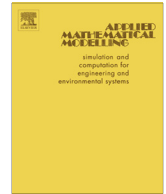




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# Dynamical properties of a biological population with a protected area under ecological uncertainty

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## ABSTRACT

In this study, stochastic differential equations with Markovian switching are used to describe the growth of some biological populations with protected areas in a random environment, where both white noise and telegraph noise are included in the ecosystem. In addition, the intrinsic growth rate and intra-specific competition coefficient are both considered to be disturbed by ecological uncertainty. We analyze the asymptotical properties of this system, such as weak persistence and extinction. Moreover, we demonstrate the biological significance and provide some real-life examples to support our main conclusions. Our results show that the establishment of a protection zone has important roles in the biological conservation process, but it is not crucial due to the presence of environmental uncertainty. We describe several theoretically strategies, which are expected to enhance the efficiency of protected areas.

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## 1. Introduction

Over-exploitation has led to the extinction of many species [1–3], so the construction of protected areas is a good management tool for protecting biological populations. Protected areas have been established as an efficient strategy for safeguarding natural biological resources and this approach was endorsed at the 2002 World Summit on Sustainable Development. Much recent research supports the establishment of protected areas, which can increase the spawning biomass, density, and mean population size (Gell and Roberts [4] and Halpern [5]). Grafton et al. [6] demonstrated that protected reserves can allow a biological population to recover faster.

Ami et al. [7] showed that protected areas can enhance biological and economic benefits. Jerry et al. [8] also demonstrated that protected areas can potentially increase the size of the viability domains during harvesting management. A mathematical model of marine reserves was produced by Loisel and Cartigny [9] to assess the impact of protected areas based on a deterministic biological population model proposed by [10]:

$$\begin{cases} dx(t) = [x(t)(a - bx(t)) + \frac{D}{h}(y(t) - x(t)) - Ex(t)]dt, \\ dy(t) = [y(t)(a - by(t)) - \frac{D}{h}(y(t) - x(t))]dt. \end{cases} \quad (1.1)$$

The living area comprises two partitions: partition  $R_1$  is the natural environment and partition  $R_2$  is a protected area. The diffusion coefficient between these two partitions is denoted by  $D$ , which is a positive constant that is assumed to be

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proportional to the density difference  $(y(t) - x(t))$ .  $H$  and  $h$  are the sizes of the natural environment and the protected area, respectively.  $E$  comprises the comprehensive negative factors in the natural area compared with the protected area, such as capture by predators and exploitation by humans.

It should be noted that the variables in model (1.1) are the population densities (biomass/size in unit area), which differs from the models proposed in proposed studies [11,12] where the population size/biomass was employed. Their corresponding model with marine reserves is as follows:

$$\begin{cases} \frac{dx_1}{dt} = rx_1 \left(1 - \frac{x_1}{sk}\right) - \sigma \left(\frac{x_1}{sk} - \frac{x_2}{(1-s)k}\right), \\ \frac{dx_2}{dt} = rx_2 \left(1 - \frac{x_2}{(1-s)k}\right) + \sigma \left(\frac{x_1}{sk} - \frac{x_2}{(1-s)k}\right) - qEx_2, \end{cases} \quad (1.2)$$

where  $x_1$  and  $x_2$  are the biomasses in the marine reserves and fishing zone, respectively, and  $s$  is the proportion of the protected area. If we let  $x_1/sk = x'_1$ ,  $x_2/(1-s)k = x'_2$  and  $E' = qE(1-s)k$ , then model (1.2) becomes

$$\begin{cases} \frac{dx'_1}{dt} = rx'_1 (1 - x'_1) - \frac{\sigma}{sk} (x'_1 - x'_2), \\ \frac{dx'_2}{dt} = rx'_2 (1 - x'_2) + \frac{\sigma}{(1-s)k} (x'_1 - x'_2) - E'x'_2. \end{cases} \quad (1.3)$$

Model (1.3) becomes a special case of model (1.1) if we denote  $sk = h$  and  $(1-s)k = H$ . Thus, the modeling principles are the same for models (1.1) and (1.3), but they are modeled from two different perspectives.

The models mentioned above are all deterministic systems. However, ecological systems are frequently affected by uncertainties, as described by May [13], Gard [14,15], and others. In this study, we consider two types of environmental factors, i.e., white noise and telegraph noise. White noise is incorporated in system (1.1) by assuming that both the intrinsic growth rate  $a$  and intra-specific competition coefficient  $b$  are affected, i.e.,

$$a \text{ changes to } a + \alpha_1 \times \text{noise one}, \quad -b \text{ changes to } -b + \alpha_2 \times \text{noise two}.$$

Thus, white noise leads to the following Itô-type system:

$$\begin{cases} dx(t) = [x(t)(a - bx(t)) + \frac{D}{h}(y(t) - x(t)) - Ex(t)]dt + \alpha_1 x(t)dB_1(t) + \alpha_2 x^2(t)dB_2(t), \\ dy(t) = [y(t)(a - by(t)) - \frac{D}{h}(y(t) - x(t))]dt + \alpha_1 y(t)dB_1(t) + \alpha_2 y^2(t)dB_2(t), \end{cases} \quad (1.4)$$

where  $B_1(t)$  and  $B_2(t)$  are standard mutually independent Brownian motions defined on a complete probability space  $(\Omega, F, P)$  with a filtration  $\{\mathcal{F}_t\}_{t \geq 0}$  that satisfies the usual conditions.  $\alpha_i$ ,  $i = 1, 2$  represents the intensity of the white noise to some extent.

Telegraph noise is introduced into the system by a finite-status  $\mathcal{S} = \{1, 2, \dots, N\}$  right-continuous Markov chain  $r(t)$ , which is also defined on the same complete probability space  $(\Omega, F, \{\mathcal{F}_t\}_{t \geq 0}, P)$ . Therefore, the system considered can be regarded as a switching between  $N$  different sub-systems. These  $N$  different systems can be caused by different survival environments, such as the food resources, rain capacity, and other factors [16,17]. The Markov chain's generation matrix  $\Theta = (\vartheta_{ij})_{N \times N}$  is given by:

$$P\{r(t + \Delta t) = j | r(t) = i\} = \begin{cases} 1 + \vartheta_{ij}\Delta t + o(\Delta t), & j = i; \\ \vartheta_{ij}\Delta t + o(\Delta t), & j \neq i, \end{cases}$$

where  $\Delta t > 0$ ,  $\vartheta_{ij} > 0$  is the transition rate from regime  $i$  to  $j$  if  $i \neq j$  and  $\vartheta_{ii} = -\sum_{i \neq j} \vartheta_{ij}$ . The Markov chain  $r(t)$  is assumed to be independent of the Brownian motions  $B_1(t)$  and  $B_2(t)$ . There are  $N$  states and system (1.4) obeys

$$\begin{cases} dx(t) = [x(t)(a(k) - b(k)x(t)) + \frac{D(k)}{h}(y(t) - x(t)) - E(k)x(t)]dt + \alpha_1(k)x(t)dB_1(t) + \alpha_2(k)x^2(t)dB_2(t), \\ dy(t) = [y(t)(a(k) - b(k)y(t)) - \frac{D(k)}{h}(y(t) - x(t))]dt + \alpha_1(k)y(t)dB_1(t) + \alpha_2(k)y^2(t)dB_2(t) \end{cases} \quad (1.5)$$

in regime  $k(k \in \mathcal{S})$ . The sizes of partitions  $R_1$  and  $R_2$  are assumed to be unchangeable because they do not change frequently in the real world. Thus, the considered system can be described by the following hybrid system

$$\begin{cases} dx(t) = [x(t)(a(r(t)) - b(r(t))x(t)) + \frac{D(r(t))}{h}(y(t) - x(t)) - E(r(t))x(t)]dt + \alpha_1(r(t))x(t)dB_1(t) + \alpha_2(r(t))x^2(t)dB_2(t), \\ dy(t) = [y(t)(a(r(t)) - b(r(t))y(t)) - \frac{D(r(t))}{h}(y(t) - x(t))]dt + \alpha_1(r(t))y(t)dB_1(t) + \alpha_2(r(t))y^2(t)dB_2(t). \end{cases} \quad (1.6)$$

System (1.6) can be regarded as a switch along  $N$  different sub-systems (1.5), where the switch depends on the status of the Markov Chain  $r(t)$ . For further information about the operational mechanism of system (1.6), please see [18] for details. The models proposed in [18] address a similar problem to that in the present study, but we assume that both the intrinsic growth rate and intra-specific competition coefficient are disturbed by ecological uncertainty. This can increase the complexity of

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