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Dynamics of a stochastic predator–prey system in a polluted environment with pulse toxicant input and impulsive perturbations

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ABSTRACT

In this study, we consider a stochastic predator–prey system in a polluted environment with pulse toxicant input and impulsive perturbations. By constructing a suitable Lyapunov function and using comparison theorem with an impulsive differential equation and stochastic differential equation, we obtain a set of sufficient conditions for extinction, with weak persistence in the mean and global attraction to any positive solution of the system. We also estimate the conditions for the upper boundedness of the expectations of this system solution. Finally, the results are verified based on computer simulations.

1. Introduction

With the rapid development of modern industry and agriculture, large volumes of toxicants and contaminants have entered the global ecosystem. Thus, environmental pollution is one of the most important socio-ecological problems. The presence of various toxicants in the environment comprises a threat to the survival of all living populations, including mankind. Therefore, it is very important to study the effects of toxicants on populations and to ascertain a theoretical threshold value that determines the persistence or extinction of a population or community. In recent years, some studies have analyzed the effects of toxicants emitted into the environment from industrial and household sources on biological species [1–3]. Most of these previous models assumed that the exogenous input of toxicants is continuous. It is known that ecological systems are often deeply perturbed by activities related to human exploitation and natural factors (e.g., harvesting, planting, drought, and flooding). In particular, sudden changes can often be characterized mathematically in the form of impulses. Therefore, the continuous input of toxicants is removed from the model and replaced by a pulse perturbation. To describe these systems more accurately, we need to employ an impulsive differential equation. The development of the theory of impulsive differential equations has led to the proposal of various population dynamical models of impulsive differential equations, which have been studied extensively [4–11]. Many important and interesting results have been obtained regarding the

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dynamical behavior of these systems, including the permanence, extinction, and global attractiveness of positive solutions and their dynamical complexity, as found in [12–14].

However, it is well known that many real-world systems may be disturbed by stochastic factors. Population systems are often subjected to various types of environmental noise (e.g., white or color noise). In ecology, it is critical to discover whether the presence of this noise has significant effects on population systems. Liu and Wang [15] considered a stochastic asymptotic predator–prey system based on the Bedding–DeAngelis functional response with stochastic perturbations. Lian and Hu [16] proposed a stochastic Gilpin–Ayala competition models and discussed the asymptotic behavior of this system. Ji et al. [17] determined the global existence of a positive unique solution of a system using the comparison theorem of stochastic equations and Itô's formula. Many important and interesting results have also been reported regarding the dynamic behavior of these stochastic systems. In particular, the stability of stochastic differential equations with impulsive perturbations has been determined based on stochastic differential equations with impulsive perturbations [18–22].

Liu and Wang [23] investigated a stochastic logistic model with impulsive perturbations and gave the sufficient conditions for the extinction, persistence in the mean, and stochastic permanence of the solution. However, few studies have addressed the population dynamics with stochastic and impulsive perturbations.

In the present study, we consider the following two-species stochastic predator–prey model in a polluted environment with a constant pulse effect on the input of toxicant and impulsive perturbations of populations:

$$\begin{cases} \left.\begin{aligned} &dx_1(t) = x_1(r_{10} - r_{11}c_0(t) - a_{11}x_1(t) - a_{12}x_2(t))dt + \sigma_{11}x_1 dB_1(t), \\ &dx_2(t) = x_2(-r_{20} - r_{22}c_0(t) - a_{22}x_2(t) + a_{21}x_1(t))dt + \sigma_{22}x_2 dB_2(t), \\ &dc_0(t) = -hc_0(t), \end{aligned}\right\} \quad t \neq t_k, \quad t \neq nT, \\ \left.\begin{aligned} &x_1(t_k^+) = (1 + a_k^1)x_1(t_k), \quad x_2(t_k^+) = (1 + a_k^2)x_2(t_k), \\ &c_0(nT^+) = c_0(nT) + p, \end{aligned}\right\} \quad t = t_k, \quad t = nT, \end{cases} \tag{1.1}$$

where $x_1(t),\ x_2(t)$ represent the population densities of the prey and predator at time t, respectively, $c_0(t)$ is the concentration of the toxicant in the organism at time $t,\ T$ is the period of the pulse effect about the exogenous input of toxicant, and p is the toxicant input amount at every time.

Throughout this study, it is assumed that:

(H_1) $r_{i0},\ a_{ii},\ r_{ii},\ \sigma_{ii},\ a_{12},\ a_{21},\ i = 1, 2$, are positive constants and
(H_2) $0 = t_0 < t_1 < t_2 < \ldots < t_k < t_{k+1} < \ldots$ are fixed impulsive points with $\lim_{k\to+\infty} t_k = +\infty,\ k \in N$, where N denotes the set of positive integers.
(H_3) When $a_k^i > 0,\ i = 1, 2$, the perturbation denotes the seeding of the species, while $a_k^i < 0,\ i = 1, 2$, represents the harvesting of the species. $\{a_k^i\}_{k=1}^{+\infty}$ is a real sequence with $a_k^i \in (-1, 1),\ i = 1, 2$.
(H_4) An integer q exists such that $t_{k+q} = t_k + T,\ a_{k+q}^i = a_k^i,\ i = 1, 2$.
(H_5) Let $(\Omega, F, \{F_t\}_{t\geqslant 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t\geqslant 0}$ that satisfies the usual conditions (i.e., it is right continuous and F_0 contains all P-null sets). Let $B_1(t)$ and $B_2(t)$ denote the independent standard Brownian motions defined on this probability space.

The remainder of this paper is organized as follows. In Section 1, we propose a stochastic predator–prey system in a polluted environment with pulse toxicant input and impulsive perturbations. In Section 2, we give some notations and lemmas. In Section 3, we obtain the sufficient conditions for the extinction, persistence in the mean of the population, and global attractiveness of system (1.1). In Section 4, we provide a brief discussion and our theoretical results are verified based on numerical simulations.

2. Preliminaries

In this section, we provide some definitions, notations, and lemmas, which are useful for explaining our main results. Next, we give some basic properties of the following subsystem of model (1.1).

$$\begin{cases} dc_0(t) = -hc_0(t), & t \neq nT, \\ c_0(t^+) = c_0(t) + p, & t = nT. \end{cases} \tag{2.1}$$

Lemma 2.1 [11]. *System (2.1) has a unique positive T-periodic solution $\widetilde{c_0}(t)$ and for each solution $c_0(t)$ of (2.1), $\lim_{t\to\infty} c_0(t) = \widetilde{c_0}(t)$. Moreover, $c_0(t) > \widetilde{c_0}(t)$ for all $t > 0$ if $c_0(0) > \widetilde{c_0}(0)$, where $\widetilde{c_0}(t) = \frac{p\exp(-h(t-nT))}{1-\exp(-hT)},\ \widetilde{c_0}(0) = \frac{p}{1-\exp(-hT)}$ for $t \in (nT, (n+1)T],\ n \in N$. From Lemma 2.1, we conclude that $\forall \varepsilon > 0$,*