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The computation of the time-dependent magnetic and electric matrix Green's functions in a parallelepiped

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ABSTRACT

A method for the computation of the time-dependent magnetic and electric matrix Green's functions in a parallelepiped with perfect conducting boundary is suggested. The method consists of the following. The equations for the magnetic Green's function are written in the form of the initial boundary value problem for a vector wave equation. Applying the Fourier series expansion approach, an explicit formula for an approximate solution of this problem is constructed. Using this formula the elements of an approximate electric Green's function are found explicitly. Numerical computation of the time-dependent magnetic and electric Green's functions has been implemented in MATLAB. The computational experiments confirm robustness of the method.

1. Introduction

The study of the Maxwell's equations in bounded domains with the natural boundary conditions is an important issue of the interdisciplinary science which has many applications in wireless telecommunications and microwave engineering [1,2]. Mobile communication systems, equipment and environment conditions are studied systematically in papers [3,4]. Nowadays the general models of electromagnetic wave propagation in closed areas are of great interest because distribution of the electromagnetic field in these environments is very complicated. The interaction of electromagnetic waves inside a real indoor environment with the rectangular geometry (cabinets, desks, etc) has been studied in [5], where the walls, ceiling and floors have been modeled as perfectly conducting.

Although the mathematical background of the theory of the initial boundary and boundary value problems for the Maxwell's equations has been described in [6] nevertheless the different aspects of correctness, such as existence, uniqueness, stability estimates of the classical and generalized statements of the initial and boundary value problems are of great interest at present [7–14]. The results of these papers have been used for constructing the analytical and numerical methods [12,15–19].

Let us mention some of them. An implicit a posteriori error estimation technique for the time-harmonic Maxwell's equations has been developed in [12]. Boulmezaoud and Rhabi in [17] have used the mortar spectral element method for the numerical solving the Maxwell's equations in a bounded domain of $\mathbf{R}^3$ with a perfect conducting boundary. In [18], Yee's scheme has been generalized to unstructured meshes, and the co-volume method has been used for the integration of the Maxwell's equations. The scattering of electromagnetic waves by perfectly conducting objects has been computed numerically and simulation of the electromagnetic wave propagation has been given.

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Analytical methods also play an important role for the construction of solutions. The Green's function technique has attracted considerable attention and is widely used for solving electromagnetic boundary value problems [5,20–27]. Most of the studies related to the Green's functions in the electromagnetic theory have been done for the frequency domain. In [5] the frequency dependent dyadic electric and magnetic Green's functions corresponding to the elementary electric and magnetic current point sources have been presented inside the cavity with perfect conducting boundary by the eigenvalues and eigenmodes. This presentation contains the solenoidal and non-divergenceless terms. In [23] the vector wave functions have been used to derive the expressions of the dyadic Green's functions. These functions were first introduced in [28]. Then [23] has applied the sets of the vector wave functions corresponding to TE (transverse electric) and TM (transverse magnetic) modes for the construction of the frequency dependent dyadic Green's functions for the rectangular waveguide and then, using the method of scattering superposition, for the construction of the frequency dependent Green's function in the rectangular cavity. The main results have been presented by quite complicated formulae and procedures for the derivation of the frequency dependent Green's functions [23]. The computation of the frequency dependent electric and magnetic Green's functions in the rectangular parallelepiped with perfect conducting boundary by the Fourier series meta-approach has been worked out in the paper [29].

Some of the approaches for the computation of the time-dependent electric and magnetic Green's functions have been developed in [30–33] for unbounded three dimensional electrically and magnetically anisotropic and bi-anisotropic media by the application of the Fourier transform with respect to space variables and matrix transformations. But the computation of the time-dependent electric and magnetic Green's functions has not been achieved so far for the bounded domain with perfect conducting conditions. We note that there exists a connection between the time dependent Green's functions and frequency dependent Green's functions which can be expressed by the Fourier transform with respect to the time variable. Moreover the time-dependent Green's function, for example, of the wave equation in whole space can be derived explicitly by the application of the inverse Fourier transform to the Green's function of the Helmholtz equation. The Fourier transform and inverse Fourier transform are defined here in the class of the generalized functions (tempered distributions) and for some scalar hyperbolic differential equations the application of the Fourier transform can be done explicitly. Unfortunately the explicit or approximate computation of the inverse Fourier transform, applied to the frequency dependent electric and magnetic Green's functions in bounded domain of the three dimensional space, is unknown. This computation requires knowledge of the frequency dependent Green's functions for all frequencies that is difficult to realize in practice. Moreover the time-dependent electric and magnetic Green's functions are singular generalized functions and numerical methods in the space of generalized functions are not developed yet. To overcome these difficulties we suggest the adaption of the Fourier series meta-approach for the direct computation of the approximate (regularized) time dependent electric and magnetic Green's functions in a rectangular parallelepiped with perfect conducting boundary which does not use the frequency dependent Green's functions. We note that the Fourier series expansion meta-approach has been applied efficiently for solving initial boundary value problems for the partial differential equations of the hyperbolic and parabolic types and the boundary value problems for the equations of the elliptic types in the bounded domains when the initial data and the inhomogeneous terms are functions from the space of differentiable functions (or Sobolev spaces). This approach has the different implementation for each type of the differential equations. In the paper [29] we have successfully adapted this approach for the approximate computation of the frequency dependent magnetic Green's function using tools of the elliptic boundary value problem solving. In the present paper we make the adaption of the Fourier series meta-approach for the approximate computation of the time dependent magnetic Green's function by means of the hyperbolic equations.

This adaption consists of the following. The equations for the magnetic Green's function are written in a special form which does not contain elements of the electric Green's function. These equations are hyperbolic partial differential equations and we applied the Fourier series expansion meta-approach for solving the initial boundary value problem for the hyperbolic partial differential equations in the bounded region. The elements of the approximate Green's function for the magnetic field are found by explicit formulae. These formulae have the form of the partial sums of the Fourier series with a finite number of terms. Using these explicit formulae we derive explicitly the approximate elements of the electric Green's matrix by integration with respect to time variable.

The simple implementation of our method for computing the Green's functions in a rectangular parallelepiped is based on the obtained presentations and does not contain any type of discretization.

1.1. The time-dependent magnetic and electric Green's functions

Let b_1, b_2, b_3 be given positive numbers, $x = (x_1, x_2, x_3)$ be a 3D variable from $\mathbf{R}^3$;

$$V = \left\{ x \in \mathbf{R}^3 : \quad 0 < x_1 < b_1, \quad 0 < x_2 < b_2, \quad 0 < x_3 < b_3 \right\}, \tag{1}$$

be a parallelepiped; Γ be the boundary of V.