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A branch-and-bound algorithm for the single machine sequence-dependent group scheduling problem with earliness and tardiness penalties

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ABSTRACT

The NP-hard single machine sequence-dependent group scheduling problem with minimization of total weighted earliness and tardiness is investigated. An arc-time-indexed formulation is presented and a Lagrangian-based branch-and-bound algorithm is proposed. The Lagrangian relaxation of the arc-time-indexed formulation is solved as a shortest path problem. The results of an extensive computational study demonstrate the efficacy of the proposed algorithm and establish characteristics of some hard to solve instances.

1. Introduction

Two popular strategies for improving the efficiency and cost-effectiveness of a production system are *just-in-time* (*JIT*) *scheduling* and *group scheduling* (*GS*). A JIT strategy improves a production system by reducing in-process inventories. A GS strategy, which schedules jobs with similar characteristics (e.g., shape or required setups) close together, reduces tooling changeovers and in-process inventories. When jobs in a group must be processed one after the other without any interruption by a job from another group, a *group technology assumption* (*GTA*) is in place. In environments where a GTA is in place, there is usually no setup time on the machine between the jobs within a group, but a setup time is incurred when the machine shifts from processing a job from one group to processing a job of another group. Sometimes the duration of the setup time depends on the previously processed group on the machine. This situation gives rise to the sequence-dependent group scheduling problem [1].

We investigate the single machine sequence-dependent group scheduling problem with earliness and tardiness penalties in which the GTA holds. By introducing earliness and tardiness penalties, we are combining group scheduling and just-in-time scheduling. The JIT requirements are represented by the earliness and tardiness penalties in the objective function. Incorporating earliness and tardiness considerations is important, because minimizing earliness and tardiness can

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significantly reduce costs. Early production increases finished goods holding cost and costs associated with contamination or deterioration of (perishable) goods. Tardy production can result in lost sales and loss of reputation. So schedules with relatively little early and late production can help a firm reduce its total cost of production and maximize its productivity and on-time delivery.

Single machine scheduling problems often appear as sub-problems in more complex scheduling problems, so focusing on a single machine scheduling problem is both academically interesting and practically relevant. In addition, results for single machine problems can provide insights that can be used to solve more complicated scheduling problems more effectively.

In the remainder, we refer to scheduling problems using the typical scheduling classification, see for example Pinedo [2]. Monma and Potts [3] study the single machine group scheduling problem with various objective functions. They show that the optimal job sequence within groups for $1|fmls, s_{hg}|\sum w_jC_j$ can be found using the weighted shortest processing time first (WSPT) rule. To determine an optimal group sequence, Ghosh [4] proposes a dynamic programming algorithm for $1|fmls, s_{hg}|\sum w_jC_j$. Hariri and Potts [5] develop a branch-and-bound algorithm for $1|fmls, s_{g}|L_{max}$. Crauwels et al. [6] propose branch-and-bound algorithms for $1|fmls, s_g|\sum U_j$. Bai et al. [7] investigate a single machine group scheduling problem in which effects of learning and deterioration are considered simultaneously. For a more extensive discussion of group scheduling problems, see Potts and Kovalyov [8].

The single machine scheduling problem with minimization of earliness and tardiness penalties has also received a fair amount of attention in the literature. Hoogeveen and van de Velde [9] propose a branch-and-bound algorithm for $1||\sum(w'_jE_j + w''_jT_j)$. Their algorithm is based on several dominance rules and various lower bounding approaches, including relaxing the machine capacity and Lagrangian relaxation. The algorithm can solve small instances with up to 20 jobs. Sourd and Kedad-Sidhoum [10] propose a faster branch-and-bound algorithm for $1||\sum(w'_jE_j + w''_jT_j)$ that can solve instances with up to 50 jobs optimally. Kedad-Sidhoum et al. [11] investigate several lower bounding methods for the single and parallel machine scheduling problem with the earliness and tardiness penalties. Recently, Sourd [12] proposes a more efficient algorithm for $1||\sum(w'_jE_j + w''_jT_j)$ with either a common due date or general due dates. Tanaka et al. [13] propose a successive sublimation dynamic programming method to solve this problem without machine idle time. Their algorithm can optimally solve instances with 300 jobs and outperforms the previous algorithms in the literature. Sourd [14] investigates the single machine earliness–tardiness scheduling problem with groups and sequence-dependent setup times and costs. However, the GTA does not have to hold. He proposes a branch-and-bound algorithm to minimize the sum of the setup costs and earliness–tardiness penalties. Subramanian et al. [15] develop an Iterated Local Search heuristic for $1|s_{ij}|\sum w_jT_j$. Tanaka and Araki [16] propose an exact algorithm for $1|s_{ij}|\sum w_jT_j$. The algorithm is an extension of their previous algorithm for the single machine scheduling problem without setup times, which is based on the successive sublimation dynamic programming method. Tanaka [17] investigates the single-machine total weighted earliness–tardiness scheduling problem with machine idle time and proposes an exact algorithm. Chang et al. [18] develop a hybrid genetic algorithm to solve the single machine scheduling problem in which a weighted sum of earliness and tardiness costs has to be minimized.

In this paper, we study the single machine sequence-dependent group scheduling problem in which the GTA is enforced and where we minimize the sum of earliness and tardiness penalties. That is, we consider a single machine and assume that N groups of jobs, each group g consists of n_g jobs, have to be processed and we seek to find a schedule for the groups as well as for the jobs within a group that minimizes the total weighted earliness and tardiness. The GTA implies that when the machine start processing one job from a group, it has to process all other jobs in that group as well before processing a job from another group. When the machine switches from processing the jobs in one group to processing the jobs in another group, a setup on the machine is required and the setup time required depends on both the group previously processed and the group to be processed. All jobs are available at the start of the planning horizon.

The problem can be denoted as $1|fmls, s_{hg}|\sum(w'_jE_j + w''_jT_j)$. Since $1||\sum(w'_jE_j + w''_jT_j)$ is NP-hard [2], it follows that $1|fmls, s_{hg}|\sum(w'_jE_j + w''_jT_j)$ is also NP-hard.

We present an arc-time-indexed formulation and a branch-and-bound algorithm. As every branch-and-bound algorithm, it has three main components: initializing, branching, and bounding. A multi-start local search algorithm is used to generate a high-quality feasible solution and initialize the search. At each node of the search tree, a bound on the value of an optimal solution is calculated using a Lagrangian relaxation bound derived from the proposed arc-time-indexed formulation. If the bound is greater than or equal to the value of the best-known solution, then the node is fathomed. Otherwise, we create two child nodes using a branching rule that exploits the structure of the solution to the Lagrangian relaxation. Innovative dominance rules are used to strengthen the Lagrangian relaxation bound.

A computational study shows that the proposed algorithm can solve instances with up to 6 groups and up to 25 jobs. The study also reveals that instances with few groups, large setup times, due dates that are spread out, and due dates that are hard to achieve (i.e., many jobs are likely to be late) are the most difficult.

The rest of the paper is organized as follows. An arc-time-indexed formulation for $1|fmls, s_{hg}|\sum(w'_jE_j + w''_jT_j)$ is proposed in Section 2. A Lagrangian relaxation method to find the lower bound is presented in Section 3. Several dominance rules for improving the Lagrangian bound and the subgradient method for maximizing the Lagrangian bound are also discussed in this section. The branching rule is discussed in Section 4. Algorithms for finding upper bounds are discussed in Section 5. Computational results are presented in Section 6. Finally, directions for future research are discussed in Section 7.