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# Multi-item fuzzy-stochastic supply chain models for long-term contracts with a profit sharing scheme

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ABSTRACT

In this study, we develop multi-item multi-objective manufacturer–retailer supply chain models with risk and budget constraints for long-term contracts with a profit sharing scheme in a fuzzy-stochastic environment. The manufacturing costs of the items are fuzzy and the demands for the items are random during each period. We examine two separate models with crisp and probabilistic budget constraints for long-term contracts where the business relationship is repeated, while demand is uncertain in a stochastic sense. The models are formulated as profit maximization problems. Next, the fuzzy compromise programming method, global criteria method, and weighted sum method are used to reduce the multi-objective problems into their corresponding single objective problems. Finally, the problems are solved using a gradient-based nonlinear programming technique: the generalized reduced gradient method. The models are illustrated with numerical examples for different types of stochastic demand and a comparative study is presented.



## 1. Introduction

In recent years, coordination and profit sharing [1–3] among members have become important for improving the system effectiveness in supply chain management research. In some markets, such as fashion or medical industry, the demand is highly unpredictable and retailers usually experience a very high level of risk. However, in most studies in this area [4–6], supply chain management problems are studied under the assumption that all supply chain members are risk neutral.

To determine the significance of risk for supply chain decisions, studies of risk-averse members have become increasingly popular in the area of supply chain management. Some researchers [7] have investigated how risk aversion affects the supply chain performance. Gan et al. [8] investigated supply chain coordination with risk-averse members under a downside risk measure. In addition, recent studies [9–11] have shown that profit sharing schemes can facilitate better coordination from the viewpoint of incentives for members. In this case, the supply chain involves the profit sharing fraction as a parameter. Under this profit sharing scheme, the retailer retains a portion of the profit that they generate and the remaining portion of the profit is shared with the manufacturer.

Some supply chain management studies have concluded that sharing information [12] among supply chain members is not sufficient to construct better supply chain contracts. In fact, better supply chain contracts involve a coordination system

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that provides incentives to all of the supply chain members, thereby allowing the decentralized and uncoordinated supply chain to behave in exactly the same manner as an integrated one (or almost) [13,14]. In general, supply contracts are designed to transfer some risk to the retailer, thereby inducing the manufacturer to have a higher capacity. Most contracts have been shown to be effective if the demand is realized after the retailer places its order. Such contracts are becoming more relevant at present in a variety of industries where the demand, or at least the demand distribution, is realized before the retailer places its order to the manufacturer, or equivalently when the retailer does not need to place an order because the manufacturer assumes this responsibility. If the final demand is realized before the retailer places its order and after the manufacturer creates its capacity, then a contract designed for the previous scenario may not be appropriate. For instance, a buyback contract is designed to handle orders that exceed demand because this situation cannot occur if the demand is realized initially. This type of contract would not be effective. In this case, the manufacturer bears the full cost of producing the capacity but does not receive the full benefit and thus they could be demotivated to invest. Therefore, an effective contract in this scenario must concentrate on the differences between the manufacturer's capacity and the final demand. From this viewpoint, a long-term contract with a profit sharing scheme is much more beneficial. The long-term contract increases the marginal benefit for the manufacturer after increasing their capacity because the additional production increases the likelihood that the extra capacity will actually be used either to meet the huge demand or as an ordering quantity in the next period, where both the manufacturer and the retailer are aware of the level of loss due to the profit sharing scheme.

In production systems, the unit manufacturing cost depends on many factors, such as the raw material cost, labor cost, wear and tear cost, and environmental protection cost [15]. Thus, if the number of units produced is increased, the labor cost per unit will be decreased. However, the wear and tear cost and environmental protection cost also increase with the number of units produced. Therefore, the unit manufacturing cost is uncertain in nature and it can be represented by an imprecise number such as a fuzzy number or a rough number. A manufacturer without sufficient previous data can represent this cost more realistically as a fuzzy number.

At present, monopoly manufacturing houses have the responsibility of creating the market for a product via advertisement, display, canvassing by sales representatives, etc., where they share part of the profit of the retailers in lieu of these activities. In addition, in the context of the current competitive market, a retailer conducts business with more than one item so the loss from one item may be compensated for by the profits from the others. In this process, the retailer experiences resource limitations, such as a limited budget, as well as the risk aversion factor.

In the present study, we formulate multi-item multi-objective supply chain models with risk and budget constraints for long-term contracts with a profit sharing scheme in a fuzzy-stochastic environment, where we consider the aforementioned real-life situations. There are two members in the supply chain: the manufacturer and the retailer. The demand is stochastic in nature and the manufacturing cost is fuzzy. Both models include the limited budget of the retailer and the amount of risk for both the manufacturer and the retailer as constraints. The maximum amounts of risk tolerance for the manufacturer and the retailer are introduced. In the first model, a crisp budgetary constraint is considered while a probabilistic budgetary constraint is introduced in the second. The models involve a long-term contract where the business relationship is repeated, while demand is uncertain in a stochastic sense. The models are structured as profit maximization problems for both the manufacturer and the retailer with risk and budget constraints. The probabilistic constraint is handled by chance-constrained programming and the credibility measure is evaluated over the fuzzy parameter. The fuzzy compromise programming method (FCPM), global criteria method (GCM), and weighted sum method (WSM) are used to reduce the multi-objective problems into their corresponding single objective problems. Next, a gradient-based nonlinear programming technique is used to solve the problems: the generalized reduced gradient (GRG) method. The models are illustrated with numerical examples for different types of stochastic demand, which follow uniform, exponential, linear, and normal distributions. A discussion based on a comparative study is also presented.

## 2. Mathematical prerequisites

### 2.1. Chance-constrained programming technique

A stochastic nonlinear programming problem with some linear chance constraints can be expressed as

$$\begin{aligned}
&\text{Max} \quad Z(x_1, x_2, \ldots\ldots\ldots, x_n),\\
&\text{s.t.} \quad \text{Prob}\left[\sum_{j=1}^{n} a_{ij}x_j \leqslant b_i\right] \geqslant r_i, \qquad (1)\\
&x_j \geqslant 0, \quad j = 1, 2, \ldots\ldots, n;\ r_i \in (0,1),\ i = 1, 2, \ldots\ldots, k,
\end{aligned}$$

where $b_i$ are normal random variables with known means and variances, and $r_i$ are specified probabilities. For simplicity, we assume that the decision variables $x_j$ are deterministic. Let $E(b_i)$ and $Var(b_i)$ denote the mean and variance of the normal variable $b_i$, respectively.

The probabilistic nonlinear programming problem stated in Eq. (1) is equivalent to the following deterministic nonlinear programming problem (cf. [16]):