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A novel variational numerical method for analyzing the free vibration of composite conical shells

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A B S T R A C T

This paper proposes an efficient numerical method in the context of variational formulation and on the basis of Rayleigh–Ritz technique to address the free vibration problem of laminated composite conical shells. To this end, the energy functional of Hamilton's principle is written in a quadratic form using matrix relations first. Displacements are then approximated via a linear combination of base functions, by which the number of final unknowns reduces. After that, the strain tensor is discretized by means of matrix differential quadrature (DQ) operators. In the next step, using Taylor series and DQ rules, a matrix integral operator is constructed which is embedded into the stiffness matrix so as to discretize the quadratic representation of energy functional. Finally, the reduced form of mass and stiffness matrices are readily obtained from the aforementioned discretized functional. To obtain the natural frequencies of conical shell, hybrid harmonic-beam base functions are employed as modal displacement functions. The accuracy of the present numerical method is examined by comparing its results with those from the published literature. It is revealed that the method is capable of accurately solving the problem with a little computational effort and ease of implementation.

1. Introduction

The study of laminated composite shell structures has gained increasing interest over the past few decades. The properties of laminated composite structures can be tailored by appropriate arrangement of the stacking sequence of the layers. Further, by combining the best aspects of the constituent layers and by appropriate choice of material for each layer, desirable properties for the whole structure can be attained. These advantages make laminated composite shell structures as promising candidates for use in various applications such as aerospace and marine industries. In some of these applications, the composite shell may be exposed to underwater or air shock loading.

Conical shells have attracted a great deal of attention from research workers these days. The analysis of the free vibration of these structures is of practical importance in structural and mechanical applications. So far, numerous studies have been carried out concerning the free vibration characteristic of isotropic and composite conical shells. Based on different shell theories, Leissa [1] summarized the equations of motion for conical shells. Several researchers used these shell theories to study the vibration of conical shells. Some notable works in this regard may include those by Saunders and his associates [2], Garnet and Kemper [3] and Siu and Bert [4] who employed the Ritz technique to investigate the free vibration frequencies.

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In addition, Irie and his associates [5,6] analyzed free vibration of conical shells with constant and variable thickness by developing a transfer-matrix scheme. The free vibration analysis of conical shells with uniform thickness was conducted by Sivadas and Ganesan [7] upon employing the finite element technique. Tong [8–10] examined the free vibration of conical shells via a simple and exact solution method. His work enabled him to study the effect of material properties on the vibrational behavior of the conical shell. The frequency characteristics of rotating conical shell were investigated by Lam and Li [11] and Li [12] by employing the Galerkin method. Liew et al. [13] studied the free vibration of thin conical shells with various boundary conditions by the use of the element-free kp-Ritz scheme. Shu [14] used the generalized differential quadrature (GDQ) method to analyze the free vibration of composite conical shells.

Development of powerful numerical approaches capable of solving problems with complicated boundary conditions is of great importance in the area of computational mechanics. One of the most efficient numerical techniques in this regard is the differential quadrature (DQ) method. This method is known to be a computationally more efficient technique compared to finite difference and finite element methods. This method was first used in the area of structural mechanics by Bert and his associates [15]. Another technique which has been extensively applied to problems arising in computational mechanics is the Ritz technique [16] which is a generalization of the Rayleigh scheme. It is a technique that depends mostly to its trial functions that must satisfy at least the essential boundary conditions. As a result, finding a proper trail function for certain boundary conditions is a major problem indeed. Among these trial functions, the products of eigenfunctions of vibrating beams, two-dimensional (2D) orthogonal plate functions and beam characteristic orthogonal polynomials have been mostly employed for 2D analyzes [17–19].

Since the analytical Rayleigh–Ritz method is performed on the energy functional, the implementation of this method necessitates the expansion of integral terms before performing the integration which involves extensive calculations. This is one of the main drawbacks of the classic Rayleigh–Ritz method. Inspired by this, developed herein is a new numerical approach in the variational form and on the basis of Rayleigh–Ritz method. In order to formulate the solution method, the energy functional of problem is first considered based on Hamilton's principle. After that, using matrix relations, the quadratic representation of energy functional is derived. Each argument of it is then approximated by a linear combination of base functions in an attempt to reduce the number of final unknowns. To discretize the strain tensor, DQ technique is applied by which a matrix differential operator is introduced. Moreover, a matrix integral operator is constructed based upon the DQ method and Taylor series. This operator is embedded into the stiffness matrix so as to discretize the quadratic form of energy functional. At last, the reduced form of mass and stiffness matrices are easily obtained from the discretized form of variational formulation. The composite conical shells are considered as the problem under study whose free vibration is investigated. Beam functions and Fourier series are employed in the axial and circumferential directions, respectively, as base functions. It can be shown that as compared to the analytical Rayleigh–Ritz method, the implementation of the present method is so much easier. One of the most crucial advantages of the present method is to bypass the integration process and expansion of energy functional which is regarded as a difficult process in the analytical Rayleigh–Ritz method, especially for large problems. Also, as the present method is directly applied to the variational form of equations, the need for the derivation of differential equations of strong form is removed. This feature can be of great significance, because the derivation of differential equations through minimizing the energy functional may be a complicated task in some cases (e.g. [20–23]). In addition, it is worth mentioning that the present formulation leads to quite symmetric domain matrices (e.g. mass and stiffness matrices). This symmetry considerably helps finding consistent solutions to the problem with a little computational cost and without decreasing the convergence rate.

2. Numerical Rayleigh–Ritz method

2.1. Fundamental concepts

Rayleigh–Ritz method is a well-known approximate scheme for the vibrational analysis of continuous systems. One major drawback of this method is that it demands extensive computations which are come from the necessity of expanding the integral terms before performing the integration of energy functional. To overcome such difficulty, a numerical variational method is established herein which facilitates the discretization of energy functional with complicated terms. To implement this method, differential and integral operators are introduced first (see Appendix A). Differential operators, which are based on the DQ rules, enable one to calculate derivatives of strain tensor in an efficient way. Further, upon employing integral matrix operators, the integration on the energy functional is eliminated. In the following, the solution methodology is described generally, and then, it is applied to analyze the free vibration of composite conical shells.

The extended Hamilton principle for a deformable body with an infinite number of degrees of freedom which occupies volume $\forall$ in the three-dimensional space denoting by $\mathbf{x} = [x_1 \quad x_2 \quad x_3]$, is given by

$$\int_{t_1}^{t_2} \delta(W + T - U)dt = 0, \tag{1}$$

where δ is the first variation, t is the time dimension, W is the external work, T is the kinetic energy and U is the elastic energy. The work done by external loads and kinetic energy are expressed as