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Frequency-dependent impedance of a strip foundation group and its representation in time domain

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ABSTRACT

A semi-analytical procedure is presented to study the dynamic cross-interference among a group of long strip foundations on an elastic half-space, which is simplified as a two-dimensional problem. In addition to analytical solutions in terms of Green functions, a discretization method is applied to determine the lateral/rocking dynamic impedances of a foundation group. To determine the unknown contact stresses between the foundations and the supporting medium, the soil-foundation interfaces are discretized into a series of strip elements which are only subjected to constant lateral and vertical tractions. Multiple foundations with different widths and separation distances are considered in the formulations. For the time history analysis of those strongly frequency-dependent impedances of a foundation group through a substructure approach, complex Chebyshev polynomials are used to fit the lumped-parameter (L-P) model in the domain-transformation. Numerical results over a wide range of vibration frequencies for impedance calculation are verified with the existing methods. The effects of cross-interference on contact stress distributions and impedances for a group of two and three rigid foundations are discussed in detail. Finally, the accuracy and the validity of the L-P model are rigorously studied by simulating adjacent foundations on an elastic half-space within a close distance.

1. Introduction

Soil-structure interaction (SSI) problem has aroused much interest in research. Various kinds of methods and outcomes have been reported in the past decades [1]. The substructure method, which enables soil and structure to be considered separately, is widely applied in the SSI. In practical situations, structures are sometimes placed close to each other due to a limitation of space. Hence, in the realistic evaluation of their dynamic characteristics, it is necessary to consider not only the interaction between the foundation and the soil medium but also the interaction between foundations through the soil, which is known as structure-soil-structure interaction (SSSI). Besides experimental investigations [2,3] and numerical procedures [4–6], substructure method is still a powerful and feasible approach to SSSI problems. The two main issues need to be addressed: (1) How to determine the impedance, i.e. the ratio of a harmonic force applied on the foundation-soil interface to the corresponding harmonic displacement for a group of strip foundations? (2) How to incorporate these frequency-dependent impedances to the substructure method for the linear/nonlinear time history analysis?

The analytical solutions for foundation impedance are obtained through a stress boundary-value method, which is based on assumptions [7–9] about the stress distributions (static rigidity, uniform, parabolic or their combinations) underneath the

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Notations

a_0	dimensionless frequency of the excitation
a_m, b_m	coefficients of the mth order of the polynomial-fraction
G_s	shear modulus of soil
H_n	lateral displacement of the nth foundation
k^∞, c^∞	stiffness coefficient and damping coefficient at the high-frequency limit
K^s	static stiffness of foundation
L_n	width of the nth foundation in the group
M	degree of the polynomial
M_n	amplitude of the rocking excitation on the nth foundation
N	number of the surface strip foundations in the group
p_n^r	uniform vertical traction applied on the rth element beneath the nth foundation
q_n^r	uniform lateral traction applied on the rth element beneath the nth foundation
Q_n	amplitude of the lateral excitation on the nth foundation
R_n	element number of interface beneath nth foundation
S_n	separation distance between the nth foundation and the $n-1$th foundation
t_m, t_m^*	the real and complex roots of the denominator of the polynomial
$\tilde{T}_m(\tilde{x})$	the mth complex Chebyshev polynomial
${}^pU_n^r(x,z)$	lateral displacement field due to the uniform vertical traction p_n^r
${}^qU_n^r(x,z)$	lateral displacement field due to the uniform lateral traction q_n^r
V_p	dilatational wave velocity of the soil
V_s	shear wave velocity of the soil
${}^pW_n^r(x,z)$	vertical displacement field due to the uniform vertical traction p_n^r
${}^qW_n^r(x,z)$	vertical displacement field due to the uniform lateral traction q_n^r
$\mathbf{X}, \mathbf{Y}$	transformation matrices
X_m, X_m^*	the real and complex residues at the pole of polynomial of impedance
Y	number of complex conjugate pole pairs
Θ_n	rocking angle of the nth foundation
ξ	wave number
ρ_s	mass density of soil
$[\Re]$	impedance matrix of the foundation group
$\Re_{mn}^{\mathrm{hh}}$	lateral impedance between nth foundation and mth foundation
$\Re_{mn}^{\mathrm{rr}}$	rocking impedance between nth foundation and mth foundation
$\Re_{mn}^{\mathrm{hr}}, \Re_{mn}^{\mathrm{rh}}$	coupling lateral-rocking impedances between nth foundation and mth foundation
γ_m, μ_m	coefficients for mth complex Chebyshev polynomial of denominator and numerator
λ_s	elastic Lamé constant
Δ_n	width of the discretized element for the nth foundation

foundation. However, as the assumed distribution may not satisfy the contact conditions at the foundation-soil interface, the deflected shape of the interface has to be modified based on the averaging techniques. More rigorous solutions for the impedance of foundation can be obtained as a mixed boundary-value problem [10–13] formulated in terms of dual integral equations. Apart from a single strip foundation, the assumed distributions of contact stresses may not be valid for adjacent or group foundations. On the other hand, as the solutions of dual integral equations cannot be expressed by elementary functions, there are limitations to obtain the contact stress functions for a SSSI problem. An approximate method based on the relaxed contact assumption was provided by Warburton et al. [14] to study the dynamic response of two rigid circular foundations on soil medium. The semi-analytical approaches with enhanced flexibility have been widely used to SSSI problems [15–20]. Wong and Luco [21] obtained the dynamic response of a system of rigid surface foundations in frequency domain by the extended boundary integral method. Savidis and Richter [22] investigated the vertical cross-interference between two rigid, massless, smooth contacting surface foundations by boundary integral equations in conjunction with constant elements in soil-foundation interface. Wang and Schmid [23] studied a similar problem with the full-space Green function. Though the full-space Green function is much simpler than the half-space one [24,25], the free soil surface around the foundation has to be discretized in addition to the discretization of foundation-soil interfaces. Dynamic interaction of arbitrary number of flexible strip foundations resting on smooth contact with a homogeneous elastic half-space was studied by Wang et al. [26] based on a classical Green function for a concentrated vertical line load. It was extended later by Senjuntichai and Kaewjuea [27] to a multilayered poroelastic half-space. Their studies were limited to vertical dynamic interaction. However, the lateral/rocking dynamic holds great significance for earthquake engineering as the largest effects of earthquake are on horizontal ground motion rather than vertical ground motion.

Soil–structure interaction analysis by the substructure method is usually applied to the frequency domain because of the frequency-dependent characteristics. The time history response of the superstructure can be obtained by using the fast