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Machine scheduling problems with a position-dependent deterioration

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ABSTRACT

In this study, we propose a new scheduling model with position-dependent deterioration, in which the processing time of a job is defined by an increasing function of total weighted normal processing time of jobs prior to it in the sequence, where the weight is position dependent. We show that some single machine scheduling problems remain polynomially solvable under the proposed model, respectively. In addition, we show that some special cases of the flow shop scheduling problems can be optimally solved by polynomial time algorithm, respectively.

1. Introduction

Scheduling models and problems are very important in the fields of manufacturing systems and service organizations. Hence numerous scheduling models and problems have been studied for many years. In classical scheduling models and problems, the processing times of jobs are assumed to be constant values. However, in many real-life situations, the job (task) processing conditions may vary over time, thereby affecting actual durations of jobs. This leads to the study of scheduling models in which the actual processing time of a job depends on its place in the sequence. There are two categories of models that address the scheduling problems with position-dependent processing times. One is related to scheduling deterioration job and the other is related to learning effects. Informally, a job processed later consumes more time than the same job when it is processed earlier, this phenomenon is known as a deteriorated effect. The skills of firms and employees continuously improve when repeating the same or similar tasks, this phenomenon is known as a learning effect. Extensive surveys of research related to scheduling deteriorating jobs and/or learning effects can be found in Alidaee and Womer [1], Cheng et al. [2], Gawiejnowicz [3], and Biskup [4], Janiak et al. [5]. More literature which has considered scheduling jobs with deteriorating jobs and/or learning effects includes Cheng et al. [6], Wang [7–9], Lee and Wu [10], Lee et al. [11], Wu and Lee [12], Wang et al. [13], Yin et al. [14], Zhang and Yan [15], Huang et al. [16], Liu et al. [17], Rudek [18,19], Wang and Wang [20], Lai et al. [21], Shen et al. [22], Wang and Wang [23], Wang et al. [24], Ji et al. [25], Lu et al. [26], Wang and Wang [27,28], Wang and Liu [29], Yin et al. [30], Yin et al. [31], Yin et al. [32], Yin et al. [33], Niu et al. [34], and Bai et al. [35].

Lee et al. [11] consider the following deterioration model:

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$$p_{jr} = p_j\left(1 + \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^a,$$

where p_j is the normal (basic) processing time of job J_j, $p_{[i]}$ denotes the normal (basic) processing time of job scheduled in the ith position in the sequence, $a \geqslant 1$ is the deterioration index and $\sum_{i=1}^{0} p_{[i]} := 0$. They proved that the makespan minimization problem can be solved in polynomial time. Wang et al. [13] considered the following deterioration model:

$$p_{jr} = p_j\left(1 + \sum_{l=1}^{r-1} p_{[l]}\right)^a,$$

where $a \geqslant 0$ is the deterioration index. They showed that the makespan minimization problem and the total completion time minimization problem for $a \geqslant 1$ can be optimally solved.

"In many real-life situations, a deterioration (aging or fatigue) effect is a phenomenon of reduction the machine efficiency as a result of its fatigue and wear. For example, in steel production, the temperature of the hot ingot on the blooming mill might drop at a slower pace as the surface cools down. The basic unit of steel making production is a charge. It refers to the concurrent smelting in the same converter. In the continuous casting stage of steel making production, the molten steel continuously solidifies into slabs at the bottom of the continuous caster, which can be regarded as a common machine. The temperature of charge to be processed at the continuous caster will drop as waiting time increases. Owing to the inevitable drop in the temperature, the processing time of charge deteriorates" (Liu et al. [17]). In this paper, we considered a new model from Lee et al. [11] and Wang et al. [13]. Specifically, we consider scheduling problems with a position-weighted deterioration, where the processing time of a job is defined by an increasing function of total weighted normal processing time of jobs prior to it in the sequence in which the weight is position dependent. The paper is organized as follows: In Section 2, we consider several single machine scheduling problems. In Section 3, we consider several flow shop scheduling problems. The last section is the conclusion.

2. Single machine scheduling problems

The single machine scheduling problem is formulated as follows. Given a single machine, there are n independent jobs $J = \{J_1, J_2, \ldots, J_n\}$. All jobs are available for processing at time 0, and the machine can handle one job at a time and job preemption is not allowed. Associated with each job $J_j (j = 1, 2, \ldots, n)$ there is a normal processing time p_j, a job-weight w_j and a due-date d_j. Let $p_{[r]}$ be the normal processing time of a job scheduled in the rth position in a sequence, and β_r be the position-dependent weight of the rth position in a sequence. Let p_{jr} be the processing time of job J_j if it is scheduled in position r in a sequence. In this paper, we consider a new deterioration model as follows:

$$p_{jr} = p_j\left(1 + \beta_1 p_{[1]} + \beta_2 p_{[2]} + \cdots + \beta_{r-1} p_{[r-1]}\right)^a, \tag{1}$$

where $a \geqslant 1$ is the deterioration index, $\beta_1 \geqslant \beta_2 \geqslant \cdots \geqslant \beta_n > 0$ and $\sum_{i=1}^{0} p_{[i]} := 0$. Obviously, the model $p_{jr} = p_j\left(1 + \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^a$, is a special case of the proposed model with $\beta_i = \frac{1}{\sum_{l=1}^{n} p_l}$, and the model $p_{jr} = p_j\left(1 + \sum_{l=1}^{r-1} p_{[l]}\right)^a$ is also a special case of the proposed model with $\beta_i = 1$.

For a given schedule $\pi = [J_1, J_2, \ldots, J_n]$. Let $C_j = C_j(\pi)$ represent the completion time of job J_j. Also let $C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}, \sum C_j, \sum C_j^\delta$ $(\delta \geqslant 0)$, $\sum L_j$ (where $L_j = C_j - d_j$ and d_j is the due date of job J_j), $\sum w_j C_j$, $\sum_{j=1}^{n} w_j(1 - e^{-\gamma C_j})$ (where $\gamma \in (0, 1)$ is the discount factor and $L_{\max} = \max\{C_j - d_j | j = 1, 2, \ldots, n\}$ represent the makespan, the total completion time, the sum of the δth power of job completion times, the total lateness, the total weighted completion time, the discounted total weighted completion time and the maximum lateness of a given permutation, respectively. Using the three-field notation scheme introduced by Graham et al. [36], our scheduling problem can be denoted as $1|p_{jr} = p_j(1 + \beta_1 p_{[1]} + \beta_2 p_{[2]} + \cdots + \beta_{r-1} p_{[r-1]})^a|Z$, where $Z \in \{C_{\max}, \sum C_j, \sum C_j^\delta, \sum L_j, \sum w_j C_j, \sum_{j=1}^{n} w_j(1 - e^{-\gamma C_j}), L_{\max}\}$.

Lemma 1. *$1 - \theta(1+t)^a + \theta a t(1+t)^{a-1} \geqslant 0$ if $a \geqslant 1, 0 < \theta \leq 1$ and $t \geqslant 0$.*

Proof. Let $h(t) = 1 - \theta(1+t)^a + \theta\, at\, (1+t)^{a-1}$. Then we have $h'(t) = \theta a(a-1)t(1+t)^{a-2} \geqslant 0$ for $a \geqslant 1, 0 < \theta \leq 1$ and $t \geqslant 0$. Hence $h(t)$ is increasing on the value of t. Since $h(t) \geqslant h(0) = 1 - \theta \geqslant 0$. This completes the proof. □

Lemma 2. *$\lambda(1 - \theta(1+t)^a) - (1 - \theta(1+\lambda t)^a) \geqslant 0$ if $\lambda \geqslant 1, a \geqslant 1, 0 < \theta \leq 1$ and $t \geqslant 0$.*

Proof. Let $f(\lambda) = \lambda(1 - \theta(1+t)^a) - (1 - \theta(1+\lambda t)^a)$. Then we have

$$f'(\lambda) = 1 - \theta(1+t)^a + \theta a t(1+\lambda t)^{a-1}$$