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Response of anisotropic thermoelastic micro-beam resonators under dynamic loads

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ABSTRACT

In this paper, the dynamic response of homogeneous, transversely isotropic, thermoelastic micro-beam resonators subjected to time-varying transverse loads has been investigated in the context of generalised theory of thermoelasticity. The micro-beam is modeled based on Euler–Bernoulli beam theory. The beam is assumed to be at clamped–clamped conditions at its axial ends. The analytical solution has been obtained by using the Laplace transform technique in the time domain. The inversion of the transformed solution has been carried out by using calculus of residues. The analytical expressions for deflection obtained in the physical domain have been computed numerically for a silicon micro-beam with the help of MATLAB software. The numerically analysed results for deflection of clamped–clamped thermoelastic silicon (Si) micro-beam with length, time and frequency ratio due to acting dynamic loads have been presented graphically. The present model may be used in micro-electromechanical applications such as relay switches, frequency filters, mass flow sensors, accelerometers and resonators.

1. Introduction

In the recent years, the field of microelectromechanical systems (MEMS) has grown rapidly and entered into many defence and communication applications. The advanced technologies for fabricating a variety of MEMS devices have been developed to meet the high demand from industries [1]. Microelectromechanical systems (MEMS) have mechanical flexible components such as micro-cantilevers, micro-bridges and micro-membranes with different geometrical dimensions and configurations that often carry load [2]. For MEMS designers, it is important to understand the mechanical properties of flexible micro-components in order to predict the amount of deflection from an applied load and vice versa so as to prevent cracking/fracture, improve performance and to increase the lifetime of MEMS devices [3]. Zener [4] explained the mechanism of thermoelastic damping and derived an analytical solution to relate the energy dissipation and the material properties of thin beam structures by assuming some mathematical and physical simplifications. Lifshitz and Roukes [5] studied the thermoelastic damping of a beam with rectangular cross sections and found that after the Debye peaks, the thermoelastic attenuation will be weakened as the size increases. Sun et al. [6] presented 2-D analysis of frequency shifts by considering heat conduction along the beam thickness and beam span by taking sinusoidal temperature gradients across the thickness of the beam. Sharma [7] derived governing equations of flexural vibrations in a transversely isotropic beam in closed form based on Euler–Bernoulli theory and studied thermoelastic damping (TED) and frequency shift (FS) of vibrations in clamped

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and simply supported beam structures. Guo et al. [8] evaluated the effect of geometry on thermoelastic damping in micro-beam resonators by using finite element method (FEM).

The dynamic response of clamped–clamped microbeams under mechanical shock has been investigated by Younis et al. [9]. Pustan and Rymuza [10] studied the mechanical properties of flexible micro-components such as micro-bridges and micro-cantilevers under mobile load by using finite element method. Yanping and Yilong [11] analysed the static deflections of micro-cantilever elastic beams under transverse loading by applying the neural network method. Pustan et al. [12] investigated the mechanical characteristics of micro-cantilevers, micro-bridges and micro-membranes under different loadings. He et al. [13] explained the non-linear response of clamped–clamped microbeam of the MEMS capacitive switch under quasi-linear or dynamic mechanical loads. Rhoads et al. [14] investigated the response of resonant micro-beams under electrostatic actuation. Choi and Lovell [15] studied the stretching effects in axially constrained doubly clamped micro-beams for mechanical and electrostatic loads by using shooting method. Sun et al. [16] investigated the vibration phenomenon during pulsed laser heating in micro-beams under different boundary conditions. Sedighi and Shirazi [17] predicted the non-linear vibration behaviour of micro-beams pre-deformed by an electric field by using Parameter Expansion Method (PEM). Jia et al. [18] studied the forced vibrations of micro-switches under combined electrostatic, intermolecular forces and axial residual stress. Alasti et al. [19] performed a study on the mechanical behaviour of a functionally gradated cantilever micro-beam subjected to a thermal moment and nonlinear electrostatic pressure. Sharma and Kaur [20] modeled and analysed the forced vibrations in micro-scale anisotropic thermoelastic beams due to time harmonic concentrated load.

Lord and Shulman [21] introduced the theory of generalised thermoelasticity (GT) with one relaxation time for isotropic materials, in which a modified law of heat conduction that includes, both heat flux and its time derivative, replaces the conventional Fourier's law. The heat equation associated with this theory is of hyperbolic type and thus eliminates the paradox of infinite speed of propagation inherent in both the uncoupled and coupled theories of thermoelasticity. This theory was extended by Dhaliwal and Sherief [22] for anisotropic materials. The existence of thermal relaxation time effect (second sound) was experimentally evidenced by Chester [23], Ackerman and Overton [24], and Ackerman et al. [25]. Achenbach [26] observed that the thermal relaxation time may be thought of as a measure of the heat conductivity of the material. Based on this observation, Dhaliwal and Singh [27] stated that the small values of thermal relaxation time correspond to highly conductive materials in which thermal disturbances travel very fast, and the large values of it refer to highly non-conductive substance to heat conduction processes.

It has been observed that several previous researchers studied the transverse vibrations, thermoelastic damping and frequency shift in micro-beams due to mechanical shocks, laser heating, electrostatic loads and moving loads. In contrast, an attempt has been made here to study the dynamic response of homogeneous, transversely isotropic, thermoelastic micro-beam resonators subjected to time-varying mechanical transverse loads. Laplace transformation technique in the time domain has been used to find the analytical expressions for deflection of the micro-beam resonator. The inversion of the transformed solution has been performed by using calculus of residues. The maximum deflection of the clamped–clamped thermoelastic micro-beam with length, time and frequency ratio under dynamic loads has been analysed numerically and presented graphically for silicon micro-beams. The effect of some related parameters on the deflection of micro-beam resonators has also been discussed.

2. Basic equations

We consider a homogenous, transversely isotropic, thermoelastic medium in Cartesian coordinate system $Oxyz$ which is initially undeformed and at uniform temperature T_0. The z-axis is taken normal to the plane of isotropy. The heat conduction equation along with constitutive relations in the context of Lord and Shulman [21] model of generalised (non-Fourier) thermoelasticity which govern displacement vector $\vec{u}(x,y,z,t) = (u_1,u_2,u_3)$ and temperature change $T(x,y,z,t) = T_1(x,y,z,t) - T_O$, in the absence of body forces and heat sources, are given by [27]:

$$K_1\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+K_3\frac{\partial^2 T}{\partial z^2}-\rho C_e\left(\frac{\partial T}{\partial t}+t_0\frac{\partial^2 T}{\partial t^2}\right)=T_0\left(\frac{\partial}{\partial t}+t_0\frac{\partial^2}{\partial t^2}\right)\left[\beta_1\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)+\beta_3\frac{\partial u_3}{\partial z}\right], \tag{1}$$

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix}=\begin{bmatrix}c_{11}&c_{12}&c_{13}&0&0&0\\ c_{12}&c_{11}&c_{13}&0&0&0\\ c_{13}&c_{13}&c_{33}&0&0&0\\ 0&0&0&c_{44}&0&0\\ 0&0&0&0&c_{44}&0\\ 0&0&0&0&0&c_{66}\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ e_{33}\\ e_{23}\\ e_{13}\\ e_{12}\end{bmatrix}-\begin{bmatrix}\beta_1\\ \beta_1\\ \beta_3\\ 0\\ 0\\ 0\end{bmatrix}T, \tag{2}$$

where σ_{ij}, e_{ij} $(i,j = 1,2,3)$ are the stress and strain tensors; t is the time and c_{ij} are the elastic parameters. Here

$$2c_{66} = c_{11} - c_{12},\ \beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3,\ \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3. \tag{3}$$

The subscripts attached to the parameters and variables denote $1 \to x$, $2 \to y$, $3 \to z$ until and unless stated otherwise. The quantities α_1 and α_3 are the coefficients of linear thermal expansion along and perpendicular to the plane of isotropy, ρ is the