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Integro quadratic spline interpolation ☆

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ABSTRACT

In this paper, we use quadratic B-splines to reconstruct an approximating function by using the integral values of successive subintervals, rather than the usual function values at the knots. It is called integro quadratic spline interpolation. Compared to the other existing methods, our method can tackle integro interpolation problem from the integral values on arbitrary successive subintervals. The general approximation error is studied and the super convergence property is also derived when the interval is equally partitioned. Moreover, it can work successfully without any boundary condition. Numerical experiments show our method is easy to implement and effective.

1. Introduction

Sometimes we deal with situations or phenomena which only involve the integral values of the function $y = y(x)$ in many practical use. The question is, if we know the integral values of the function then how we can use this information to reconstruct the approximating function [1].

Suppose that the interval $[a, b]$ is partitioned into the following n successive subintervals $[x_i, x_{i+1}], i = 0, 1, \dots, n-1$, where $a = x_0 < x_1 < \cdots < x_n = b$.

Assume that the function values $y_i = y(x_i)$ are not given, but the integral values I_i of $y(x)$ are known on the n subintervals $[x_i, x_{i+1}]$. Our task is to determine an integro-interpolating function $S(x)$ such that

$$\int_{x_i}^{x_{i+1}} S(x)dx = I_i = \int_{x_i}^{x_{i+1}} y(x)dx, \quad i = 0, 1, \dots, n-1.$$

This problem has many practical applications in the fields of mechanics, statistics, climatology, oceanography, numerical analysis and so on. Spline functions, as the piecewise polynomials with certain smoothness at the knots, can be used to tackle this new interpolation problem. There are several research papers addressing this issue. In 1996, Behforooz [1] firstly introduced a new method to construct integro cubic splines by using the integral values of $y(x)$, rather than the usual function values at the knots.The method was deduced by using cubic Hermite interpolation polynomial formula together with three additional boundary conditions. Later, an integro quintic spline approach was discussed by Behforooz [2],which is primarily based on the quintic Hermite-Birkhoff interpolation polynomial. Unfortunately, these method are expected to face a more complicated construction process and these method need several additional boundary conditions besides the

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given integral values. Moreover, they did not study the derivatives approximation of $y(x)$. To overcome this drawbacks, Zhanlav and Mijiddorj [3] discussed the local integro cubic splines and their approximation properties in 2010. It was able to reconstruct $y^{(k)}(x), (k = 0, 1, 2)$ with $O(h^{4-k})$ errors respectively. But its error orders were relatively lower. In 2012, Lang and Xu [4] discussed the integro quartic spline interpolation by using quartic B-splines and this method possesses super convergence orders in approximating function values and second-order derivative values at the knots.Very recently, Wu and Zhang [5] discussed the integro sextic spline interpolation and its super convergence. However, all the existing methods are used to tackle integro spline interpolation from given integral values of successive uniform subintervals since they used the uniform B-spline bases. That is to say, they always assume $x_i = a + ih$, for $i = 0, 1, \ldots, n$ with $h = (b - a)/n$.

In this paper, we discuss the integro interpolation problem by using quadratic B-splines. Our new method has some advantages compared to the other existing methods. We highlight them as follows.

(I) Our method can tackle the integro interpolation problem from the integral values of $y(x)$ on arbitrary subintervals $[x_i, x_{i+1}], i = 0, 1, \ldots, n-1$.
(II) Our method is easier to implement. The degree of spline function is lowest and it mainly requires to solve a three-band linear system with $n - 1$ equations.
(III) Our method possesses super convergence order in approximating function values at the knots and also performs well without any additional boundary condition when the interval $[a, b]$ is equally partitioned. That is to say, $S(x)$ can approximate $y(x)$ with $\mathcal{O}(h^4)$ errors at the equidistant nodes.

The rest of this paper is organized as follows. In Section 2, we present integro quadratic spline interpolation and analyze its error estimate. In Section 3, we modify the integro quadratic spline interpolation without any additional boundary condition when the interval $[a, b]$ is equally partitioned. Section 4 is devoted to numerical experiments, numerical results show that our proposed method is effective. Finally, we conclude our paper in Section 5.

2. Integro quadratic spline interpolation

2.1. Construction of integro-interpolating quadratic splines

The problem of integro-interpolating quadratic spline is addressed as follows.

Suppose $S(x) \in C^1[a, b]$ and $S(x)$ is a quadratic polynomial on each subinterval $[x_i, x_{i+1}]$, where $a = x_0 < x_1 < \cdots < x_n = b$, then $S(x)$ is called a quadratic spline function with respect to the given knots $x_0, x_1, \ldots, x_n$. If given the integral values I_i of $y(x)$ of each subinterval $[x_i, x_{i+1}]$ and satisfying

$$\int_{x_i}^{x_{i+1}} S(x)dx = I_i = \int_{x_i}^{x_{i+1}} y(x)dx, i = 0, 1, \ldots, n-1, \tag{1}$$

then $S(x)$ is called an integro-interpolating quadratic spline function.

Now, we are going to use these numbers I_i to construct this class of integro quadratic spline functions $S(x)$. The quadratic spline $S(x)$ is a piecewise quadratic polynomial such that $S(x)$ and $S'(x)$ are continuous on $[a, b]$. The basic idea is to construct the quadratic polynomial from the three parameters $S(x_i), S(x_{i+1})$ and I_i. For simplicity, we use the following notations: $S_i = S(x_i)$ and $S_{i+1} = S(x_{i+1})$.

From a simple computation, $S(x)$ can be defined by

$$S(x) = S_i H_i(x) + I_i H_{i,i+1}(x) + S_{i+1} H_{i+1}(x), x \in [x_i, x_{i+1}], \tag{2}$$

where,

$$H_i(x) = \left(1 - 3\frac{x - x_i}{x_{i+1} - x_i}\right)\left(\frac{x - x_{i+1}}{x_i - x_{i+1}}\right),$$
$$H_{i,i+1}(x) = \left(\frac{6}{x_{i+1} - x_i}\right)\left(\frac{x - x_{i+1}}{x_i - x_{i+1}}\right)\left(\frac{x - x_i}{x_{i+1} - x_i}\right),$$
$$H_{i+1}(x) = \left(1 - 3\frac{x - x_{i+1}}{x_i - x_{i+1}}\right)\left(\frac{x - x_i}{x_{i+1} - x_i}\right).$$

Obviously, $S(x)$ is continuous on the interior nodes $x_1, x_2, \ldots, x_{n-1}$. Meanwhile, we have

$$S'(x_{i+1} - 0) = \frac{-6}{h_i^2} I_i + \frac{2}{h_i}(S_i + 2S_{i+1}), h_i = x_{i+1} - x_i.$$

Similarly, we have the expression of $S(x)$ on $[x_{i+1}, x_{i+2}]$ and can compute $S'(x_{i+1} + 0)$. Certainly, we have the continuity conditions for $S'(x)$, i.e.,

$$S'(x_{i+1} - 0) = S'(x_{i+1} + 0).$$