Applied Mathematical Modelling 39 (2015) 3050–3062

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An exact solution for vibrations of postbuckled microscale beams based on the modified couple stress theory

R. Ansari *, M.A. Ashrafi, A. Arjangpay

Department of Mechanical Engineering, University of Guilan, P.O. Box 3756, Rasht, Iran

ARTICLE INFO

Article history:
Received 15 June 2013
Received in revised form 4 November 2014
Accepted 17 November 2014
Available online 27 November 2014

Keywords:
Microbeam
Free vibration
Postbuckling
Modified couple stress theory

ABSTRACT

The modified couple stress theory, as a theory capable of capturing size effects, is implemented to study the vibration characteristic of a postbuckled microbeam. To this end, a modified couple stress Euler–Bernoulli beam model containing geometric nonlinearity is considered. Within the framework of a variational formulation and based on Hamilton's principle, the governing equation and corresponding boundary conditions are derived. By eliminating time-dependent terms, the governing equation of vibration is reduced to that of buckling problem for the microbeam subjected to an axial load. The critical buckling loads and their corresponding mode shapes are predicted through an exact solution for various boundary conditions. Afterwards, the vibration analysis of a simply-supported microbeam is investigated around the obtained postbuckling configuration. It is found that the stiffness of microbeam predicted by the modified couple stress model is higher than that predicted by the classical model. Additionally, it is demonstrated that the natural frequencies by considering all of the vibration modes except the first mode are independent of the buckling load. The influences of the dimensionless length-scale parameter, Poisson's ratio, various boundary conditions and the number of buckled modes on the critical buckling loads and natural frequency are fully investigated.

1. Introduction

Recently, microbeams have attracted a lot of interest due to their applications in sensors, actuators, micro- and nano-electromechanical systems [1–4]. The size-dependent deformation and vibrational response of microstructures have been demonstrated experimentally [5,6]. Tang and Alici [7] estimated the micro- and nano-sized length-scale factors using experimental data collected from nanoindentation and microindentation experiments and incorporated them into the natural frequency and static-deflection models. They also presented the effect of the length-scale factor on the stiffness of the cantilevers [8]. Liu et al. [9] experimentally investigated the size effects on the torsional response of micro-sized polycrystalline copper wires by using a novel automated torsion balance. Liu et al. [10] also studied the plasticity of micron scale metallic wires under cyclic torsion by using the torsion balance technique.

In order to predict the size-dependent behavior of small scale structures, the non-classical elasticity theories have been developed, as the conventional theories are scale-free and not capable of explaining the size-dependent phenomena. Some of these size-dependent elasticity theories are strain gradient theories [11,12], nonlocal elasticity theories [13,14] and couple stress theories. The classical couple stress theory was initially proposed by Touplin [15], Mindlin and Tiersten [16] and Koiter

* Corresponding author. Tel./fax: +98 131 6690276.
E-mail address: r_ansari@guilan.ac.ir (R. Ansari).

http://dx.doi.org/10.1016/j.apm.2014.11.029
0307-904X/

[17] in 1960s in which two length-scale parameters are employed in the constitutive equation in addition to the classical Lame constant. To simplify the equations of the couple stress theory, Yang et al. [18] introduced the modified couple stress theory (MCST) including only one additional length-scale parameter for a specific material. By utilizing this theory, Park and Gao [19] analyzed the bending of Euler–Bernoulli beam model capturing the size effect. Furthermore, Kong et al. [20] performed a study on the dynamic problems of Euler–Bernoulli microbeams analytically and found the significant difference between natural frequencies predicted by the MCST and the classical model when the characteristic sizes such as thickness and diameter are small. Additionally, the microstructure-dependent Timoshenko beam model was formulated by Ma et al. [21] in which the static bending and free vibration problems were solved directly using the MCST for a simply-supported beam. More recently, Asghari et al. [22] delineated the nonlinear size-dependent static bending of a Timoshenko beam model, which was solved numerically, and predicted the free vibration behavior of the beam using the method of multiple scale. In addition, Asghari et al. [23] presented an analytical solution based on the MCST to investigate the size-dependent linear bending and free vibration of a functionally graded Euler–Bernoulli beam. They found that a functionally graded (FG) Euler–Bernoulli beam based on the MCST is stiffer than a similar Timoshenko beam. Ke et al. [24] employed the differential quadrature method together with an iterative algorithm to determine the nonlinear vibration frequencies of the FG microbeams based on the MCST and von Karman geometric nonlinearity. Wang et al. [25] presented an analysis of the nonlinear free vibration of microbeams based on the MCST in which the equations were solved numerically by applying the shooting method. They found that the size effect is significant for small ratios of the beam height to material length-scale. Şimşek et al. [26] investigated the static bending and free vibration of simply-supported FG microbeams by developing the Navier-type solution. Moreover, Şimşek et al. [27] developed the static bending analysis of a microscale FG Timoshenko beam model in which the governing equations were solved analytically for a simply-supported beam subjected to point and uniformly distributed loads. They concluded that the deflections of the microbeam by the classical beam theory are always larger than those by the MCST. Akgoz and Civalek [28] presented bending analysis of micro-sized beams based on the Bernoulli–Euler beam theory within the modified strain gradient elasticity and MCSTs. In addition, Akgöz and Civalek [29] investigated the static bending and free vibration behavior of simply-supported microbeams by developing a new size-dependent higher-order shear deformation beam model according to the modified strain gradient theory. Nateghi et al. [30] investigated the buckling analysis of FG microbeams based on the MCST for three different beam theories with various boundary conditions by applying generalized differential quadrature method. Solving the nonlinear buckling problem due to considering the geometric nonlinearity for a given axial load yields to the postbuckling configuration. Nayfeh and Emam [31] presented an exact solution for the nonlinear buckling problem of Euler–Bernoulli beams. They also calculated the natural frequencies around all of the buckled configurations. Nagai et al. [32] presented the experimental results on the chaotic vibrations of a post-buckled beam subjected to periodic lateral acceleration. Xia et al. [33] developed the static, postbuckling and free vibration analysis of a nonlinear Euler–Bernoulli beam model in accordance with the MCST in which the critical buckling loads were obtained analytically and the free vibration behavior was predicted using the multiple scales method. Akgöz and Civalek [34] investigated the buckling behavior of size-dependent FG microbeams for different boundary conditions on the basis of Bernoulli–Euler beam and modified strain gradient theories. They also presented the differences between the results obtained by the modified strain gradient model and those predicted by modified couple stress and classical continuum models. Lestari and Hanagud [35] presented analytical solutions for the dynamics of buckled beams with different types of end conditions. Asadi et al. [36] analytically investigated the free vibration of shape memory alloy hybrid composite beams in thermally pre/post-buckled domains. Moradi and Jamshidi Moghadam [37] studied vibration analysis of cracked post-buckled beam and extracted the natural frequencies and mode shapes of the cracked beam. Ansari et al. [38] analyzed the thermal postbuckling characteristics of microbeams made of functionally graded materials based on modified strain gradient theory through the generalized differential quadrature method in conjunction with a direct approach without linearization. Farokhi et al. [39] investigated the nonlinear resonant behavior of a microbeam over its buckled configuration by employing MCST. Ansari et al. [40] presented an exact solution for the thermal postbuckling and vibration analyses of FG microbeams based on MCST.

In the present study, the vibration behavior of a postbuckled Euler–Bernoulli microbeam based on the MCST is investigated. The Von Karman strain tensor is employed to consider the geometric nonlinearity and Hamilton's principle is applied to obtain the governing equations and associated boundary conditions. The buckling problem is solved by using an exact solution for microbeams with various boundary conditions in order to obtain the critical buckling loads and their corresponding mode shapes. Then, the vibration behavior of the microbeam is analyzed around the obtained postbuckling configuration with simply supported-simply supported end conditions. The influence of the vibration mode numbers and the critical buckling load on the obtained natural frequency is investigated. In addition, the effects of geometrical parameters, Poisson's ratio, various boundary conditions and buckling modes on the critical buckling loads and natural frequency are fully examined.

2. Formulation of the microbeam

2.1. Modified couple stress theory

In accordance with the modified couple stress theory (MCST) proposed by Yang et al. [18], the deformation measures, $\boldsymbol{\epsilon}$ and $\boldsymbol{\chi}$, conjugate to $\boldsymbol{\sigma}$ and $\boldsymbol{m}$, are introduced as the following relations. The infinitesimal strain tensor $\boldsymbol{\epsilon}$ is defined by