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An interpolating boundary element-free method for three-dimensional potential problems

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ABSTRACT

This paper begins by discussing an improved interpolating moving least-square (IIMLS) method and the properties of its shape function. In the IIMLS method, the shape function is of delta function property and the weight function is nonsingular, so it overcomes the drawbacks in both the moving least-square approximation and the interpolating moving least-square method. Then combining boundary integral equations with the IIMLS method, an interpolating boundary element-free method (IBEFM) is developed for three-dimensional potential problems. In the IBEFM, only a nodal data structure on the boundary face of a domain is required. Unlike the boundary node method (BNM), the IBEFM is a direct meshless method in which the primary unknown quantities are real solutions of nodal variables, and boundary conditions can be imposed directly and easily, which leads to a greater computational precision. Besides, the number of both unknowns and system equations in the IBEFM is only half of that in the BNM, and thus the computing speed and efficiency are increased. Numerical examples on curve/surface fittings and potential problems indicate that the efficiency and convergence rate of the present methods is high.

1. Introduction

The finite element method (FEM) and the boundary element method (BEM) have been the dominant numerical computational methods in the field of computational science and engineering for several decades. Both methods depend on the generation of meshes, adapted or not. Mesh generation in some situations is still arduous, time consuming and fraught with pitfalls. To circumvent the problems associated with meshing, meshless (or meshfree) methods to obtain numerical solutions of boundary value problems without resorting to an element frame have been developed in the past two decades.

The moving least-square (MLS) [1] is an approximation method, developed by Lancaster and Salkauskas, which generates continuous functions from a cluster of unorganized sampled point values based on the computation of a weighted least squares approximation. Li and Zhu [2,3] obtained error estimates of the MLS approximation in Sobolev spaces. Since the numerical approximations of the MLS start from scattered nodes instead of meshes, there have many meshless methods based on the MLS scheme. Among them are the diffuse element method, the element-free Galerkin (EFG) method [4], the *h-p* meshless method [5], the meshless local Petrov–Galerkin (MLPG) method [6], the moving least square reproducing kernel method [7,8], the reproducing kernel hierarchical partition of unity [9,10], and so on. These meshless methods followed the idea as the FEM, in which the problem domain is discretized.

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The MLS approximation has also been used in boundary integral equations (BIEs). Typical of them are the meshless local boundary integral equation (LBIE) method [6] and the boundary node method (BNM) [11–13]. As the BEM, these BIEs-based meshless methods have emerged as promising numerical techniques in scientific computing. Nonetheless, since the MLS approximations lack the delta function property of the usual FEM and BEM shape functions, boundary conditions in these meshless techniques are difficult to be implemented. The technique used in the BNM involves a new definition of the discrete norm used for the construction of the MLS approximations, which doubles the number of unknowns and system equations. Accordingly, the advantage of BIEs-based meshless methods is therefore eroded and discounted to a certain extent. via combining a variational form of BIEs and the MLS approximations, another technique is developed in the symmetric Galerkin BNM to impose boundary conditions [14–17]. However, in all these BIEs-based meshless methods, the basic unknown quantities are approximations of nodal variables, and therefore they are indirect meshless methods. Recently, Liew et al. [18] developed an improved MLS scheme that uses weighted orthogonal polynomials as basis functions. The improved MLS scheme has been introduced into BIEs to develop a direct meshless method, the boundary element-free method (BEFM) [18,19]. Because the improved MLS scheme still lacks the delta function property, boundary conditions in the BEFM are implemented with constraints [20,21].

To restore the delta function property of the MLS, Lancaster and Salkauskas further developed an interpolating moving least-square (IMLS) method that uses specific singular functions as weight functions [1]. Based on the IMLS scheme, an improved EFG has been proposed by Kaljevic and Saigal [22]. Besides, by revising the formulae of the IMLS method and combining it with BIEs, Ren et al. proposed an interpolating boundary element-free method (IBEFM) [20,21] and an interpolating element-free Galerkin (IEFG) method [23,24] for two-dimensional potential and elasticity problems. In these improved schemes, boundary conditions can be imposed as easily as in the FEM and the BEM. A drawback of the IMLS method is that the involved weight function is singular at nodes. The singularity complicates the computation of the inverse of the singular matrix and thus reduces the computing speed and efficiency. To avoid this shortcoming, Netuzhylov [25] introduced a perturbation technique into the IMLS method. This method requires a new parameter ε, which is used for regularization of the weight function matrix. Although the parameter will affect the performance of this method, there is still no theoretical strategy to get appropriate values for this parameter.

In order to overcome problems in both the MLS approximation and the IMLS method, Wang et al. [26,27] proposed an improved interpolating moving least-square (IIMLS) method, and an improved interpolating EFG method and an IBEFM for two-dimensional potential problems. The IIMLS method has the following features: (1) the shape function possesses the delta function property, so implementing boundary conditions in the IIMLS-based meshless method is much easier than that in the MLS-based meshless method; (2) the weight function used is nonsingular at any point, hence any weight function used in the MLS approximation can also be used in the IIMLS method; and (3) the number of unknown coefficients in the trial function of the IIMLS method is less than that of the MLS approximation, thus fewer nodes are required in the influence domain and a higher computational accuracy can be achieved in the IIMLS method.

In this paper, we first discuss the IIMLS method and the properties of its shape function. Specially, the reproducing properties of the IIMLS shape function is proved theoretically and verified numerically in detail. The capability of data fittings of the IIMLS method for one- and two-dimensional functions is also studied. Then, a new implementation of the interpolating boundary element-free method (IBEFM) is developed for boundary-only analysis of three-dimensional potential problems. In our implementation, the IBEFM combines the IIMLS for construction of interpolation functions with BIEs for the governing partial differential equations. Besides, the IIMLS interpolants have been suitably truncated at corners and edges in order to avoid the discontinuity of the IIMLS shape function. Since potential problem is one of the most important problems in mathematics which has wide applications to a number of topics relevant to mathematical physics and engineering, the current IBEFM is formulated for three-dimensional potential problems [28]. In the IBEFM, the boundary of the problem domain is discretized via properly scattered nodes. Since the IIMLS shape functions satisfy delta function properties, the IBEFM overcomes the shortcomings of the BNM. The enforcement of boundary conditions in the IBEFM is as easy as in the conventional BEM. Therefore, the number of both system equations and unknowns in the IBEFM is only half of that in the BNM.

The rest of this paper is outlined as follows. Section 2 presents the IIMLS method and the properties of its shape function. Curve and surface fittings as examples are also studied in this section to show the capacity and accuracy of the IIMLS method. In Section 3, a detailed numerical implementation of the IBEFM is described. A comparison between the IBEFM, the BNM and the BEM is also included in this section. Then, numerical results are given in Section 4. Section 5 contains conclusions.

2. The improved interpolating moving least-squares (IIMLS) method

2.1. Notations

Let D be a bounded domain in $\mathbb{R}^d$ $(d = 2,3)$. A general point of $\mathbb{R}^d$ is denoted as $\mathbf{x} = (x_1, x_2, \dots, x_d)$ or $\mathbf{y} = (y_1, y_2, \dots, y_d)$. For any $\mathbf{x} \in D$, assume that the influence domain of $\mathbf{x}$ is a d-dimensional ball $\Re(\mathbf{x})$ with radius $r(\mathbf{x})$, i.e.,

$$\Re(\mathbf{x}) = \{\mathbf{y} : \|\mathbf{x} - \mathbf{y}\| \leqslant r(\mathbf{x})\},$$

where $\|\cdot\|$ may denote any norm in $\mathbb{R}^d$. For simplicity, $\|\cdot\|$ is the Euclidean norm.

Besides, given $\mathbf{x} \in D$, assume that there have n nodes $\mathbf{x}_i$ such that $\mathbf{x} \in \Re_i,\ i = 1, 2, \dots, n$, where $\Re_i = \Re(\mathbf{x}_i)$ is the influence domain of $\mathbf{x}_i$.