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Short communication

Improved delay-dependent exponential stability criteria for neutral-delay systems with nonlinear uncertainties

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ABSTRACT

In this paper, the problem of delay-dependent exponential stability criteria for neutral-delay system with nonlinear uncertainties is considered. By constructing a new class of Lyapunov–Krasovskii functionals and using the free-weighting matrices methods within a convex optimization approach, some less conservative delay-dependent stability criteria of neutral delay differential system with nonlinear uncertainties is obtained. These stability conditions are solved as linear matrix inequalities which can be easily solved by various convex optimization algorithms. Finally, numerical examples are given to illustrate the effectiveness of the results.

1. Introduction

In recent decades, considerable attention has been devoted to the time delay systems due to their extensive applications in practical systems including circuit theory, chemical processing, bio engineering, neural networks, automatic control and so on. Since time delays may cause oscillation, divergence or instability, stability analysis for time delayed systems has become a topic of great theoretic and practical importance in recent years. In particular, in some practical systems [1,2], delay exists not simply in the state, but also in the derivatives of the state. This class of time delay systems are referred to as neutral delay differential system. It turns out that stability of neutral delay systems proves to be a more complex issue because the systems involve the derivative of the delayed state. On the other hand, nonlinear uncertainties are commonly encountered since it is very difficult to obtain an exact mathematical model as result of environmental noise, slowly varying parameters, and so on. In [1,2], basic dynamic properties of neutral systems were presented. Some results on robust stability and control of time-delay systems were provided in [3–6]. Improved stability analysis of time-delay systems with nonlinear uncertainties was treated in [7,8]. For classes of neutral time-delay systems, the subject of robust stability were examined in [9–13] where some related criteria were derived. With focus on the nature of delays in neutral systems, some delay-dependent stability results and linear matrix inequality criteria were established in [14–24]. H_∞ filtering for linear neutral systems with mixed time-varying delays and nonlinear perturbations was analyzed in [25].

The results mentioned in the literatures [1–25] are only concerned with the asymptotic stability. However, the exponential stability problem is also important since it can determine the convergence rate of system states to equilibrium points. In

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[26], the problem of exponential stability criterion for time delay systems with nonlinear uncertainties was studied. Zhang et al. [27] investigated the problem of neutral switched systems with interval time-varying mixed delays and nonlinear perturbations by employing the Lyapunov–Krasovskii functional technique and the free-weighting matrix method. Based on the generalized eigenvalue problem approach, the exponential stability criteria for neutral delay differential system with nonlinear uncertainties was derived in [28]. However, these results have conservatism to some extent, which exist room for further improvement.

Motivated by the statement above, in this paper, the problem of delay-dependent exponential stability criteria for neutral delay differential system with nonlinear uncertainties is considered. A new class of Lyapunov–Krasovskii functional is constructed to derive some novel delay-dependent stability criteria. The obtained criteria are less conservative due to Jensen integral inequality, delay-partitioning method, free-weighting matrices method and a convex optimization approach. These stability conditions are implemented as linear matrix inequalities which can be easily solved by various convex optimization algorithms. Three numerical examples are given to illustrate the effectiveness of the proposed method.

Notations. The notations in this paper are quite standard. I demotes the identity matrix with appropriate dimensions, R^n denotes the n dimensional Euclidean space, and $R^{m\times n}$ is the set of all $m \times n$ real matrices, $\|\cdot\|$ stands for the Euclidean norm of given vector. $*$ denotes the elements below the main diagonal of a symmetric block matrix. For symmetric matrices A and B, the notation $A > B$ (respectively, $A \geqslant B$) means that the matrix $A - B$ is positive definite (respectively, nonnegative), $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$ mean the largest and smallest eigenvalue of given square matrix, respectively. $diag\{\ldots\}$ denotes the block diagonal matrix. $\|\varphi\| = sup_{-h\leqslant s\leqslant 0}\{\|\phi(s)\|, \|\dot{\phi}(s)\|\}$, where h is some positive constant.

2. System description and preliminaries

Consider the following neutral delay differential system with nonlinear uncertainties

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\dot{x}(t-h(t)) + f_1(t,x(t)) + f_2(t,x(t-h(t))) + f_3(t,\dot{x}(t-h(t)), \\ &x(t) = \phi(t), \dot{x}(t) = \dot{\phi}(t), t \in [-h, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state, $\phi(t)$ is continuously differentiable function on $[-h, 0]$. A, B and C are known real constant matrices. $h(t)$ is the time-varying delay satisfying

$$0 \leqslant h(t) \leqslant h, \quad 0 \leqslant \dot{h}(t) \leqslant d \leqslant 1,$$

where $h$and $d$are positive constants. $f_1(t,x(t))$, $f_2(t,x(t-h(t))$ and $f_3(t,\dot{x}(t-h(t))$ are nonlinear uncertainties and are assumed to satisfy the following conditions,

$$\|f_1(t,x(t))\| \leqslant \alpha_1\|x(t)\|,$$

$$\|f_2(t,x(t-h(t)))\| \leqslant \alpha_2\|x(t-h(t))\|,$$

$$\|f_3(t,\dot{x}(t-h(t)))\| \leqslant \alpha_3\|\dot{x}(t-h(t))\|,$$

where α_1, α_2 and α_3 are known constants.

The following lemmas will be used for providing the main results in the sequel.

Definition 2.1. The system (2.1) is said to be globally exponentially stable if there exist scalars $k > 0$ and $\gamma > 0$ such that for every solution $x(t)$ of (2.1)

$$\|x(t)\| \leqslant \gamma\|\phi\|e^{-kt}.$$

Lemma 2.2 (*The integral inequality, Gu et al. [29]*). *For any constant matrix, $M \in R^{n\times n}$, $M = M^T > 0$, scalars $\gamma_1 < \gamma(t) < \gamma_2$, and a vector function $x : [-\gamma_2, -\gamma_1] \to R^n$ such that the following integrations concerned is well defined, then*

$$-(\gamma_2 - \gamma_1)\left(\int_{t-\gamma_2}^{t-\gamma_1} \dot{x}^T(s)M\dot{x}(s)ds\right) \leqslant -[x(t-\gamma_1) - x(t-\gamma_2)]^T M[x(t-\gamma_1) - x(t-\gamma_2)].$$

Lemma 2.3 (*Schur complement lemma [30]*). *Given constant matrices X, Y, Z with appropriate dimension satisfying $X = X^T$, $Y = Y^T$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0, \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

Lemma 2.4. *For any vectors $a, b \in R$, and scalar $\epsilon > 0$, we have*

$$2a^T b \leqslant \epsilon a^T a + \epsilon^{-1} b^T b.$$