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Single machine common flow allowance scheduling with deteriorating jobs and a rate-modifying activity

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ABSTRACT

We study a single machine slack due date assignment (usually referred to as SLK) scheduling problem with deteriorating jobs and a rate-modifying activity. The deterioration effect manifest such that the job processing time is a function of its starting time in a sequence. The rate-modifying activity is an activity that changes the processing rate of machine, i.e., the machine performs a rate-modifying activity. Hence the actual processing time of a job is a variable, which depends not only on its starting time in a sequence but also on whether it is scheduled before or after a rate-modifying activity. The goal is to schedule the rate-modifying activity, the optimal common flow allowance and the sequence of jobs to minimize the total earliness, the total tardiness and the common flow allowance cost. We show that the problem remains polynomially solvable under the proposed model.

1. Introduction

In traditional scheduling problems, the processing times of jobs are fixed constant. This assumption, however, does not allow adequate modeling of realistic industrial processes. Hence, in recent years, there is a growing interest in scheduling problems involving *deteriorating jobs*, i.e., jobs whose processing times are increasing functions of their starting times. Extensive surveys of different scheduling models and problems involving jobs with start time dependent processing times (deteriorating jobs) can be found in Gawiejnowicz [1] and Rustogi and Strusevich [2]. More recent papers that have considered scheduling problems with deteriorating jobs include Cheng et al. [3], Lee et al. [4], Lee and Lai [5], Cheng et al. [6], Lee and Lu [7], Sun et al. [8], Lai and Lee [9], Wang and Wang [10], Wang and Wang [11], Yang [12], Wang et al. [13], Cheng et al. [14], Lee [15], Oron [16] and Xu et al. [17].

On the other hand, machine may become unavailable due to a machine breakdown or preventive maintenance during the scheduling period. Lee and Leon [18] first studied scheduling problems with a rate-modifying activity, i.e., after the rate-modifying activity, the machine's production efficiency can be improved. Machine scheduling with a rate-modifying activity can be considered as a special type of scheduling with maintenance. An extensive survey of different scheduling problems and models involving machine availability constraints can be found in Ma et al. [19]. More recent papers which have considered scheduling jobs with machine availability constraints include Mosheiov and Sarig [20], Zhao et al. [21], Mosheiov and Sidney [22], Yang et al. [23], Lodree and Geiger [24], Wang and Wang [25], Wang et al. [26], Zhao et al.

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[27], Hsu et al. [28], Zhao and Tang [29], Mor and Mosheiov [30], Cheng et al. [31], Rustogi and Strusevich [32], Hsu et al. [33], Yang and Yang [34], Wang et al. [35], Rustogi and Strusevich [36], Ji et al. [37] and Yin et al. [38].

In addition, many scheduling researchers considered various due date assignment scheduling problems (see [39,40]), only a few focused on the so-called flow allowance (usually referred to as slack (SLK) method). In this paper, we extend the SLK due date assignment model proposed by Adamopoulos and Pappis [41] to the case where there are both deteriorating jobs and a rate modifying maintenance activity. The goal of this study is to find the optimal common flow allowance, the optimal sequence, and the optimal position of the rate modifying activity to minimize a total cost function that dependents on the earliness, the tardiness and the common flow allowance. The remainder of this study is organized as follows: In the second section we formulate the problem. In the third section, we introduce an optimal polynomial time algorithm to solve the problem. In the last section we conclude the paper and suggest some topics for future research.

2. Problem description

There are n jobs $J = \{J_1, J_2, \ldots, J_n\}$ and a single machine simultaneously available at time zero for processing. The machine may be assigned to perform a maintenance activity, which will change its production rate and last t_0 units of time. During the maintenance activity, no production is performed. Let p_j and $\delta_j p_j$ $(0 < \delta_j \leqslant 1)$ be the normal processing time of job J_j $(j = 1, 2, \ldots, n)$ if it is scheduled before and after the maintenance activity, where δ_j is the modifying rate of job J_j. Let s_j be the starting time of job J_j, as in Zhao and Tang [29], we assume that the processing time of a job deteriorates linearly in relation to its start time, i.e., the actual processing time of job J_j if it is scheduled before and after the maintenance activity is given by $p_j^A = p_j + bs_j$ and $p_j^A = \delta_j p_j + bs_j$, where b is the common deterioration rate of all jobs. As in Adamopoulos and Pappis [41], we assume that the due date $d_j = p_j^A + q = p_j + bs_j + q$ and $d_j = p_j^A + q = \delta_j p_j + bs_j + q$ if the job is scheduled before and after the maintenance activity, $j = 1, 2, \ldots, n$, where q is the common flow allowance (is a decision variable). Let $C_j, E_j = \max\{0, d_j - C_j\}$ and $T_j = \max\{0, C_j - d_j\}$ be the completion time, the earliness and the tardiness of job J_j, $j = 1, 2, \ldots, n$, respectively. Similar to Wang and Wang [25], we aim to determine the optimal common flow allowance q, the optimal location of a rate modifying activity, and the optimal schedule such that the total cost function

$$Z = \sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma q) \tag{1}$$

is minimized, where α, β and γ denote the unit penalties for earliness, tardiness and common flow allowance, respectively. Let rm denote a rate modifying activity, following the standard notation of Graham et al. [42], the problem can be denoted as $1|rm, p_j + bs_j|\sum(\alpha E_j + \beta T_j + \gamma q)$.

3. Main results

For convenience, we denote that the rate modifying activity is scheduled in position i $(0 \leqslant i \leqslant n)$ if it is scheduled immediately after the job scheduled in the ith position. Let $[i]$ denote the job scheduled in the ith position of a sequence, for C_j, we have

$$\begin{aligned}
&C_{[0]} = 0,\\
&C_{[j]} = C_{[j-1]} + p_{[j]} + bC_{[j-1]} = p_{[j]} + (1+b)C_{[j-1]} = \sum_{l=1}^{j} p_{[l]}(1+b)^{j-l}, \quad j = 1, 2, \ldots, i,\\
&C_{[i+1]} = C_{[i]} + t_0 + \delta_{[i+1]}p_{[i+1]} + b(C_{[i]} + t_0) = C_{[i]}(1+b) + t_0(1+b) + \delta_{[i+1]}p_{[i+1]},\\
&C_{[j]} = C_{[j-1]}(1+b) + \delta_{[j]}p_{[j]}, \quad j = i+2, \ldots, n.
\end{aligned} \tag{2}$$

For s_j we have

$$\begin{aligned}
&s_{[1]} = 0,\\
&s_{[i+1]} = C_{[i]} + t_0\\
&s_{[j]} = C_{[j-1]}, \quad 2 \leqslant i \leqslant n, \ j \neq i+1.
\end{aligned} \tag{3}$$

The actual processing time of job $p_{[j]}$ is:

$$\begin{aligned}
&p_{[j]}^A = p_{[j]} + b\sum_{l=1}^{j-1} p_{[l]}(1+b)^{j-1-l}, \quad j = 1, 2, \ldots, i,\\
&p_{[j]}^A = \delta_{[j]}p_{[j]} + b\sum_{l=1}^{i} p_{[l]}(1+b)^{j-1-l} + b\sum_{l=i+1}^{j-1} \delta_{[l]}p_{[l]}(1+b)^{j-1-l} + b(1+b)^{j-1-i}t_0, \quad j = i+1, i+2, \ldots, n.
\end{aligned} \tag{4}$$