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Two-product inventory management with fixed costs and supply uncertainty

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ABSTRACT

This paper determines the optimal ordering policy for a two-product, periodic-review inventory problem in which the probability of supply availability is unknown. Moreover, there are two different fixed costs assigned to each product. Demand rates are random variables with known probability density functions, and the supply availability for each product is updated at the beginning of each time period. We prove the optimality of (s,S) policy with a monotone switching curve that indicates the priority of production, where the order-up-to levels and the reorder points are functions of supply availability information. A simple computation is proposed to calculate the two threshold levels. Bayesian updating helps to manage the optimal ordering policy by updating supply disruption information. Numerical results show that improving the accuracy of the forecast leads to making a better ordering decision and eliminating the negative effect of supply disruption on the total cost.

1. Introduction

There are predictable and unpredictable factors that cause supply disruptions, which negatively impact the performance of a supply chain and its members, such as uncertainty of demand and supply (e.g., [1,2]). Demand uncertainty has been studied by many researchers in the field of production planning, inventory and supply chain management. Supply uncertainty has received considerable attention in the literature recently. This interest is because it is: (1) an important topic and (2) not many studies are available in the literature. Traditional production and inventory policies set by firms to encounter the risks that arise from supply disruptions may be ineffective [3]. A firm can learn from its experience with its supplier, for instance, the history of supply availability, to determine how reliable that supplier is. Through observations, a firm can acquire additional information that leads to more accurate ordering decisions.

This paper addresses the optimal ordering/production scheduling policy for a system that produces two products with finite planning horizon, where inventory is reviewed periodically due to variation in demand and uncertainty in supply availability. Two switching costs are incurred, for example, when the facility switches production between two products. The developed model adopts a structure of supply availability of all-or-nothing; i.e., in a given period the supply is either fully available or unavailable. The model is used to determine the optimal ordering/production policy for each of the two

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products. The developed model is also applicable for periodic production planning problems with unreliable manufacturing facilities.

There is a plethora of published works in the literature on inventory with probabilistic and uncertain demand; however, it would not be effective and feasible to review them here. Thus, the following review focuses on the ordering/production planning literature with fixed costs, uncertain supply, and (s,S) inventory policy. The review of the literature classifies the works on inventory/production planning problem into two groups. These include studies that considered fixed cost(s) and those that considered uncertainty of supply. The earliest works of the first category are those of [4,5] who dealt with the concept of K-convexity in stochastic inventory management, for a single product case where a K-convex function is defined as for any $0 < \alpha < 1$, $K \geqslant 0$ (fixed cost), and $x \leqslant y$, $f(\alpha x + (1-\alpha)y) \leqslant \alpha f(x) + (1-\alpha)(f(y) + K)$. They showed that a periodic review (s,S) inventory policy for a single product is optimal under certain assumptions. Johnson [6] investigated a multi-product periodic review inventory problem with a single fixed cost for all products, with an order cost for each. For the infinite planning horizon case, Johnson [6] showed that the (σ,S) policy is optimal. This policy operates as follows, if the inventory level of a product at the beginning of a period is in the reordering region, σ, then order up to S otherwise do not order. Kalin [7] redefined σ as a do-not-order set, an increasing set, regarding a specific partial ordering. He considered a single product with a fixed cost and recommended an optimal order policy that when the initial inventory level is in the set σ do-not-order, else order up to the vector level S. Elhafsi and Bai [8] studied a production and setup scheduling problem in which a machine produces two products with fixed setup times and costs for both. Under the assumption of constant demand rate and constant processing time for each product, the optimal production rate and setup epochs are derived as a function of the system's state for the cases of finite and infinite time horizons.

To compute the optimal ordering policy, Ohno and Ishigaki [9] presented a time-computing-efficient algorithm, the Policy Iteration Method (PIM), to minimize the expected undiscounted cost function for multi-item continuous time inventory model with fixed replenishment time. Shaoxiang [10] showed that the optimality of the hedging point policy is based on the concept of μ differential monotone for finite and infinite planning horizons, for a two-product periodic review stochastic inventory system with no setup (fixed) costs, where demands are random and the production facility is unreliable. A function, $G_n(x,y)$, is said to be μ-differential monotone if: (1) $G'_{nx^+}(x,y)$ when $\uparrow x$ and $\uparrow y$, (2) $G'_{ny^+}(x,y)$ when $\uparrow x$ and $\uparrow y$, and (3) $\mu_1 G'_{nx^+}(x,y) - \mu_2 G'_{ny^+}(x,y)$ when $\uparrow x$ and $\downarrow y$, where, e.g., $\uparrow x$ is non-decreasing in x and $\downarrow y$ is decreasing in y ([10]; p. 314). A generalization of the K-convexity in R^n was proposed by [11] for the case of joint and individual setup costs to determine the optimal policy for a multi-product inventory problem. In addition to the definition of K-convexity in R^n, some properties of the function were developed. Numerically, they showed that the (σ,S) policy is optimal for a two-product, two-period inventory problem with deterministic demand and a joint setup cost. Chen and Simchi-Levi [12] investigatd periodic review of stochastic cash balance problem when there are fixed costs for both ordering and return. The objective was to make ordering or return decisions to minimize the total expected cost for N-period planning horizon. To characterize the optimal policy, they developed the concept of symmetric K-convexity and (K, Q)-convexity.

Demirag et al. [13] developed two heuristic policies for a firm in which an ordering (fixed) cost is incurred when the order quantity of the previous period does not cross a specified threshold level. They partially characterized an optimal policy that is simpler and easier for practical implementation. In a follow-up paper, Demirag et al. [14] determined the optimal policy for a single-product stochastic periodic review inventory problem operating under three different forms of the fixed cost: (1) If the order size exceeds a threshold value C, then a fixed cost K_2 is incurred, otherwise $K_1 < K_2$, (2) an incremental value of K is incurred per batch for any additional order quantity higher than batch capacity C, and (3) an additional fixed cost is incurred for any order as well as a fixed cost charged for all batches. They introduced an optimal policy for a new concept of $(C, K_1 + K_2)$-convexity for (1); an X–Y band type optimal policy applies for (2) and (3).

For continuous review of the capacitated inventory/production problem, Chao et al. [15] developed an (r,S) policy and presented an algorithm to compute parameters r and S. They assumed that demand arrivals follow a Poisson distribution and setup costs are incurred with every production run. Their optimal production policy suggests turning the machine on to produce, if the inventory level of a certain product is below r, and off if it is above S.

The second group of studies of interest focuses on reviewing stochastic inventory/production planning where product supply availability is uncertain. The review first considers those studies that deal with where the supply distribution is known. Parlar et al. [16] showed that the (s,S) policy is optimal and a reorder point, s, is dependent on the previous supply state, and that, s, increases if the current period of supply is fully filled. To compute the optimality of order-up-to level policy, Güllu et al. [17] presented a newsboy-like formula for a single-product, periodic review inventory order-up-to level policy with supply uncertainty for the cases of deterministic and dynamic demand. They assumed that supply availability follows a Bernoulli process and supply to be at given time either available in full or unavailable. Özekici and Parlar [18] considered a periodic review inventory–production planning control problem with and without fixed costs in an environment in which the cost parameters and demand and supply availability are randomly changing. A base-stock policy and an (s,S) policy were shown to be optimal when a supplier is unreliable.

The aforementioned papers consider supply uncertainty with perfect knowledge on supply distribution. Azoury [19], Lariviere et al. [20] and Scarf [21] are examples of the studies that deal with demand Bayesian learning and its effect on production decisions. On the contrary, few are those works on reducing supply uncertainty by learning. To the best of the authors' knowledge, there are two papers that address dual-sourcing and inventory management when supply distribution is unknown and supply uncertainty is forecasted by the Bayesian learning approach. Tomlin [3] was the first to investigate