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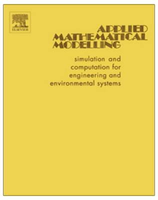

A meshless numerical approach based on Integrated Radial Basis Functions and level set method for interfacial flows ☆

L. Mai-Cao [a], T. Tran-Cong [b],*

[a] Faculty of Geology and Petroleum Engineering, Ho Chi Minh City University of Technology, Viet Nam
[b] Computational Engineering and Science Research Centre, Faculty of Health, Engineering and Sciences, The University of Southern Queensland, Toowoomba, QLD 4350, Australia

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ABSTRACT

This paper reports a new meshless Integrated Radial Basis Function Network (IRBFN) approach to the numerical simulation of interfacial flows in which the two-way interaction between a moving interface and the ambient viscous flow is fully investigated. When an interface between two immiscible fluids moves, not only its position and shape but also the flow variables (i.e. velocity field and pressure) change due to the presence of surface tension along the moving interface. The velocity field of the ambient flow, on the other hand, causes the interface to move and deform as a result of momentum transport between the two immiscible fluids on both sides of the interface. Numerical investigations of such a two-way interaction is reported in this paper where the level set method is used in combination with high-order projection schemes in the meshless framework of the IRBFN method. Numerical investigations on the meshless projection schemes are performed with typical benchmark incompressible viscous flow problems for verification purposes. The approach is then demonstrated with the numerical simulation of two bubbles moving, stretching and merging in an incompressible ambient fluid under the action of buoyancy force.

1. Introduction

Fluid flows studied in this paper with a moving interface between two immiscible fluids can be classified as interfacial flows. In general, a numerical approach to the simulation of such flows consists of (a) a flow modelling method, (b) an interface modelling algorithm, and (c) a flow-interface coupling technique. These three components should be coupled together in a consistent framework in order to properly model complicated phenomena associated with the interfacial flows.

Regarding the flow modelling, there are two main approaches to formulating the governing equations for an interfacial flow: one-fluid and two-fluid models. In the one-fluid model, a single flow equation is formulated to describe both fluid flows, and a characteristic function is used to specify a particular fluid [1]. In the two-fluid model, on the other hand, each fluid has its own governing equations and therefore the characteristics of each phase can be separately captured [2]. For incompressible interfacial flows, the governing equations in either one-fluid or two-fluid model are formulated from the

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* Corresponding author. Tel.: +61 7 4631 1332; fax: +61 7 4631 2110.
E-mail addresses: maicaolan@hcmut.edu.vn (L. Mai-Cao), thanh.tran-cong@usq.edu.au (T. Tran-Cong).

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Navier–Stokes equations (NSEs). Among others, the projection/pressure correction method can be used to solve NSEs. First proposed in [3], the projection method consists of a predictor–corrector procedure in which the momentum equation is first solved using an initial approximation of the pressure to obtain an intermediate velocity field. A pressure correction is then obtained by solving a Poisson equation. Finally, the new velocity field is updated using the intermediate velocity and the pressure correction. Several improvements to the original projection method have been made by (a) improving intermediate velocity boundary conditions [4]; or (b) improving accuracy order in time via pressure correction procedure [5]; or (c) improving pressure boundary conditions [6]. In this work, a class of new meshless projection schemes is developed based on the improved projection methods mentioned above to solve flow equations in the one-fluid model.

Numerical approaches to interface modelling can be classified in two groups: moving-grid and fixed-grid methods. For the moving-grid methods, the interface is treated as the boundary of a moving surface-fitted grid [7]. This approach allows a precise representation of the interface whereas its main drawback is the severe deformation of the mesh as the interface moves. The second approach which is based on fixed grids includes tracking and capturing methods. The tracking methods explicitly represent the moving interface by means of predefined markers [8]. In capturing methods, on the other hand, the moving interface is not explicitly tracked, but rather captured via a characteristic function. Examples of the capturing methods are phase field method [9], volume-of-fluid method [10] and level set method [11]. The characteristic function used to implicitly describe the moving interface is the order parameter in the phase field method, volume fraction in the volume-of-fluid method and level set function in the level set method. For these capturing methods, no rezoning/remeshing is needed to maintain the overall accuracy even when the interface undergoes large deformation.

Regarding flow-interface coupling in the numerical simulation of interfacial flows, the surface tension is normally taken into account in the computation of force balance at the interface where the difference in stresses of the two fluids in the direction normal to the interface is balanced by the surface tension on the interface [7]. A simple and effective model that alleviates the interface topology constraints was presented in [12] where the proposed model, known as the continuum surface force (CSF) model, interprets the surface tension as a continuous, three-dimensional effect across an interface rather than as a boundary value condition on the interface. The advantage of the CSF model is that the moving interface needs not be explicitly described for the interfacial boundary condition.

In this work, all of the aforementioned modelling techniques are implemented within the meshless framework of the IRBFN method for interfacial flows. The idea of using radial basis functions for solving partial differential equations (PDEs) was first proposed in [13,14] to solve parabolic, hyperbolic and elliptic PDEs. Since its introduction, various methods based on radial basis functions have been developed and applied in different areas. A local radial point interpolation method (LRPIM) was proposed in [15] for free vibration analyses of 2-D solids. The Linearly Conforming Radial Point Interpolation Method (Lc-Rpim) was studied in [16] for solid mechanics. The IRBFN method has been reported to be a highly accurate tool for approximating functions, their derivatives, and solving differential equations [17,18]. The method was then successfully applied to transient problems [19] including those governed by parabolic as well as hyperbolic PDEs where comparisons of performance of the IRBFN-based methods and others, including finite difference, boundary element and finite element methods, were made. Additionally, high-order meshless schemes have been implemented for passive transport problems in [20] where the motion and deformation of moving interfaces in an external flow are fully captured by a unified numerical procedure combining the IRBFN method and the level set method together with the semi-Lagrangian method or Taylor series expansions. Furthermore, two numerical meshless schemes were proposed in [21] for the numerical solution of Navier–Stokes equations. Based on the projection method and coupled with high-order time integration techniques in the meshless framework of the IRBFN method, the two schemes showed their good stability and accuracy when applied to the numerical simulation of incompressible fluid flows and interfacial flows in [21]. In the present work, the two schemes are further numerically investigated, specifically for Navier–Stokes equations with time-dependent boundary conditions as well as for the numerical simulation of buoyancy-driven bubble flow. In comparison with finite difference or finite element methods, the unique advantages of the proposed IRBFN-based methods include (i) the meshless nature and implicit interface capturing technique of the present approach; and (ii) the ability to effectively ensure the conservation of mass during the evolution of the interfaces between fluids.

The remaining of this paper is organized as follows. Firstly, the one-fluid model is formulated for interfacial flows with an introduction to the CSF model and level set method. The new meshless projection schemes for the one-fluid continuum model are then presented followed by the step-by-step procedure of the proposed meshless approach to interfacial flows. Numerical investigations on the new projection schemes with typical viscous flows as well as the application of the proposed meshless IRBFN-based approach to the numerical simulation of two bubbles moving, stretching and merging in an ambient viscous flow are then performed for verification purposes.

2. Mathematical formulation

Consider a domain Ω and its boundary $\partial\Omega$ containing two immiscible Newtonian fluids, both being incompressible. Let Ω_1 be the region containing fluid 1 at time t. Similarly, let Ω_2 be the region containing fluid 2 and bounded by the fluid interface Γ at time t. The governing equations describing the motion of the two fluids in their own regions are given by the Navier–Stokes equations,