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Amplitude control and projective synchronization of a dynamical system with exponential nonlinearity

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ABSTRACT

A dynamical system with exponential nonlinearity is reported in this paper. Analysis shows that one can flexibly control signal amplitude of the system by introducing control function in the exponential nonlinear term, and the corresponding Lyapunov exponents keep invariable. But the coefficient in cross-product term cannot provide amplitude modulation. By considering the fact that the unique amplitude function can provide the scale factors, a control scheme combining the techniques of linear feedback and variable substitution is presented to realize projective synchronization of the chaotic system.

1. Introduction

As an interesting phenomenon presented in nonlinear dynamical system, chaos is deterministic, random-like, unpredictable and sensitive to initial conditions [1–8]. The now-classic Lorenz system [1] is accounted a landmark for chaos investigation and has motivated a great deal of interest and design of 3D autonomous chaotic systems with simple mathematic formula, such as Rölsser system [2], Chen system [3], Lü system [4], Liu system [5], Yang system [6], Qi system [7], and Kim system [8].

Recently, a class of chaotic system with both amplitude control and constant Lyapunov exponent has been constructed, whose chaotic behavior is achieved by absolute value nonlinearity [9–11]. Subsequently, another class of chaotic system with amplitude control and constant Lyapunov exponent has been proposed, in which the smooth nonlinear term is of the essence to generate chaos [12–15]. The feature of adjustable amplitude of a chaotic system is important in applications. From an implementation point of view, it turns out to obtain the desired signal amplification without any extra circuitry spending and prevent from increasing the probability of failure in circuit operation. Meanwhile, this feature can avoid the influence of the band-limit filter in signal amplification circuit. Therefore, it is a promising type system to provide a new security encoding key in chaotic radar and chaotic communication.

For the amplitude control of smooth system with quadratic nonlinear terms, the fundamental principle is that, by a judicious linear scale transformation of the variables, the variation of the coefficients of quadratic nonlinearities leads to a set of differential equations that can be degenerated to the original system of equations. Matter of fact, when the quadratic term is the only nonlinearity, the corresponding coefficient can control the signal amplitude since it uniquely determines the scale of the variables. As an illustrative example, we consider the following chaotic system with a single quadratic nonlinearity as $\dot{x}_1 = -0.2x_2$, $\dot{x}_2 = x_1 + x_3$, $\dot{x}_3 = x_1 - x_3 + x_2^2$ [16]. This system can be described by $\dot{x}_1 = -0.2x_2$, $\dot{x}_2 = x_1 + x_3$,

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$\dot{x}_3 = x_1 - x_3 + kx_2^2$ with the linear transformation $x_1 \to kx_1, x_2 \to kx_2, x_3 \to kx_3$. Therefore, coefficient of the sole nonlinear term x_2^2 can scale the amplitude of x_1, x_2, x_3 according to $1/k$. Spontaneously, one will ask whether each coefficient of quadratic term can provide amplitude control? This question will be addressed in this study.

Chaos synchronization can be explained as that the state responses of the controlled dynamical system verge on those of other system. Since Pecora and Carroll originally proposed the drive-response concept for achieving synchronization between two chaotic systems [17,18], various synchronization schemes have been introduced, such as complete synchronization [19,20], anti-synchronization [21], phase synchronization [22], generalized synchronization [23], projective synchronization [24,25] and Q-S synchronization [26], etc. For the existed synchronization schemes, it's particularly worth recalling the works given in [19,20], in which the authors realized the complete synchronization of chaotic systems by combining robust technique with time delay estimation, and the proposed scheme is simple since the control inputs are added to some of the differentiate equations of the slave system. The dynamical behavior of projective synchronization is that the time trajectories of the master and slave systems synchronize up to a constant proportional factor. This characteristic is often used to extend binary digital communication to M-nary for achieving faster response [24]. Generally, to realize projective synchronization with another proportional factor between master and slave systems, the corresponding scaling factor in the controller needs to be reselected [24,25].

In this paper, we report a dynamical system with exponential nonlinear term. Differing from the present smooth system with constant Lyapunov spectrum [12–15], the unique parameter for amplitude control is the coefficient of the exponential nonlinear term, and the coefficient of cross-product term cannot provide amplitude modulation. A practical scheme for amplitude modulation of chaotic signals is considered by introducing a control function. Detailed analysis shows that one can control signal amplitude of the chaotic system flexibly with the selecting of control function, while the corresponding Lyapunov exponent spectrums keep invariable. By considering the control function of amplitude modulation as the inherent scale factor, a simple and flexible method is achieved to realize projective synchronization of the chaotic system. The synchronization scheme is developed based on the control techniques of single linear feedback [27] and single-variable substitution [28], which is more practical compared with the works given in [19,20]. What's more, the proposed synchronization scheme will open up a magnificent prospect for engineering application since we can discretionarily set the scale of the signals between the drive and response systems by selecting the amplitude function in the synchronization systems.

2. Dynamical system with exponential nonlinear term

The introduced dynamical system with exponential nonlinear term is expressed as follows:

$$\begin{cases} \dot{x}_1 = -ax_1 + x_2 \\ \dot{x}_2 = bx_2 + x_1x_3 \\ \dot{x}_3 = -x_3 - e^{x_2^2} \end{cases}. \tag{1}$$

Here x_1, x_2, x_3 are the state variables, and a, b are the positive parameters. When $a = 10$, $b = 8$, the three Lyapunov exponents of system (1) is calculated as 1.541755 > 0, 0.000341, −5.869741 < 0. The Kaplan–Yorke dimension of the system is $D_{KY} = 2 + (1.541755 + 0.000341)/5.869741 = 2.2627$, revealing that the Kaplan–Yorke dimension is fractional. Therefore, system (1) is indeed chaotic. The corresponding chaotic phase diagrams and Poincare mapping on plane $x_3 = -100$ are depicted in Fig. 1. It appears from Fig. 1 that the reported system displays abundantly complicated behaviors of chaotic dynamics.

System (1) has two equilibrium points, which are respectively described as below:

$$E_+\left(\sqrt{\ln ab}/a,\ \sqrt{\ln ab},\ -ab\right),\quad E_-\left(-\sqrt{\ln ab}/a,\ -\sqrt{\ln ab},\ -ab\right).$$

And the characteristic equation is given by:

$$|\lambda I - J| = -\lambda^3 + [-a + b - 1]\lambda^2 + [ab - a + b + x_3 + 2x_1x_2e^{x_2^2}]\lambda + ab + x_3 + 2ax_1x_2e^{x_2^2}, \tag{2}$$

when $a = 10$, $b = 8$, the two equilibrium points are calculated as $E_+(0.662, 2.0933, -80)$, $E_-(-0.662, -2.0933, -80)$. And they hold the same characteristic roots, as

$$\lambda_1 = 17.2263,\quad \lambda_2 = -10.1131 + 5.1412i,\quad \lambda_3 = -10.1131 - 5.1412i.$$

Here, λ_1 is a positive real number, λ_2 and λ_3 become a pair of complex conjugate roots with negative real parts. Obviously, the two equilibrium points are saddle-focus point with two-dimensional stable manifold and one-dimensional unstable manifold.

The three Lyapunov exponents and the divergence of system (1) is

$$\sum_{i=1}^{3} LE_i = \nabla V = \partial\dot{x}_1/x_1 + \partial\dot{x}_2/x_2 + \partial\dot{x}_3/x_3 = -a + b - 1. \tag{3}$$

System (1) would be dissipative and converges to a subset of measure zero at an exponential rate of e^{-a+b-1} when $-a + b - 1 < 0$, this means that for an initial volume V_0, the volume will become $V(0)e^{(-a+b-1)t}$ at time t through the flow generated by the system. Therefore, all system orbits are ultimately confined to a specific subset with zero volume and the asymptotic motion settles onto an attractor.