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A modified truncated Newton algorithm for the logit-based stochastic user equilibrium problem

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ABSTRACT

In this paper, we investigate Newton type algorithms to solve the path-based logit stochastic user equilibrium problem in static traffic assignment. This problem is in essence an equality constrained minimization problem. Traditionally, we can first use the variable reduction method to transform this problem into an unconstrained one, and then apply the truncated Newton method to solve this unconstrained minimization problem. All procedures are performed in the reduced variable space. However, we do not know a priori which variables are appropriate to be chosen as reduced variables. If the reduced variables are poorly chosen, sometimes the computation times may be unacceptably high, due to the ill conditioning of the reduced Newton equation.

To overcome this drawback, we propose a modified truncated Newton (MTN) algorithm. Compared to the traditional algorithm above, MTN has three distinct features: (1) The iteration point and search direction are generated in the original variable space. (2) The Newton equation is approximately solved in the reduced variable space. (3) The reduced variables can be changed from one iteration to another. These features make MTN particularly suitable for the path-based SUE problem and exhibits superlinear convergence. In order to efficiently solve the reduced Newton equation in each iteration, we propose a principle on how to select the reduced variables. We compare the MTN algorithm with the Gradient Projection (GP) algorithm on the Sioux Falls and Winnipeg networks. The GP algorithm is one of the most efficient algorithms that currently exists for solving the problem. We show that with suitable settings, our MTN algorithm outperforms the GP algorithm, in particular in network problems with high levels of congestion and where the route choice is more deterministic (i.e., more sensitive to route costs), which are the hardest traffic assignment problems to solve. We therefore conclude that our MTN algorithm is currently the fastest algorithm for solving such complex traffic assignment problems.

1. Introduction

The stochastic user equilibrium (SUE) static traffic assignment problem has been extensively studied in the past decades. This problem was defined first by Daganzo and Sheffi [1], who generalized the user equilibrium (UE) principle of Wardrop

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[2]. The stochastic user equilibrium principle states that no traveler can reduce his or her own perceived travel time by unilaterally changing routes. Among various stochastic models proposed in the literature, the most popular models are the probit model and the multinomial logit model.

Under the assumption that users' perceived path times follow a multivariate normal distribution, the probit SUE model results. Daganzo and Sheffi [1] first formulated probit SUE as a generalization of user equilibrium. Sheffi and Powell [3] showed that the probit SUE model could be formulated as a mathematical programming problem. Maher and Hughes [4] extended Sheffi and Powell' model, and proposed an interesting unconstrained minimisation model for the elastic demand probit SUE problem. Meng et al. [5] built a linearly constrained minimisation model and a solution algorithm for the probit SUE problem with fixed demand, link capacity constraints and separable link travel time functions. This model was extended in Meng et al. [6] to the case of elastic demand and nonseparable link travel time functions.

Alternatively, assuming that users' perceived path times follow a multivariate Gumbel distribution yields the multinomial logit-based SUE model. Fisk [7] showed that the multinomial logit SUE model can be formulated as an equivalent mathematical programming formulation. Fisk's model has the appeal of being relatively close to the probit model while keeping a convenient analytical closed form. Fisk's model has received significant attention, not only with respect to theoretical investigations [8–12], but also for practical implementations [13–15]. However, it is well known that the multinomial logit SUE model has some drawbacks. It cannot properly deal with path overlap and perception variance with respect to paths of different lengths. To overcome the deficiencies of the multinomial logit SUE model, some modification models had been proposed in the literature. These extended logit model include C-logit [16], path size logit [17], cross-nested logit [18], paired combinatorial logit (PCL) [19], generalized nested logit [20], and logit kernel [21]. All these logit-type models are closed form, but the multinomial logit SUE model is the foundation of others. In this study, we will focus on a solution algorithm based on the multinomial logit model, which is abbreviated as logit model in the remainder of this paper.

In general, solution algorithms for the logit based SUE problem can be divided into two categories. In the first category are link-based algorithms, which do not require explicit path enumeration. It only assumes an implicit path choice set, such as the use of all efficient paths [11,22], or all cyclic and acyclic paths [10,23]. In the other category are path-based algorithms, which require explicit choice of a subset of feasible paths prior to the assignment. Therefore, it makes the path choice set more realistic from a behavioral standpoint. In the literature, a large number of path choice set generation methods are proposed by different authors, see for example Ben-Akiva et al. [24], Azevedo et al. [25], De la Barra et al. [26], and Cascetta et al. [27]. Some path set generation methods are constrained deterministic enumeration methods using so-called branch-and-bound decision rules to add routes to the choice set (e.g. [28,29]), while others are stochastic methods which repeatedly add shortest paths to the choice set using randomized link costs (e.g. [30]).

Path-based algorithms for solving the SUE problem are mainly discussed in Bekhor and Toledo [31]. In their study, they proposed using the Gradient Projection (GP) method presented by Bertsekas [32]. In each iteration, the gradient of the objective function is projected on a linear manifold of the active constraints, with the scaling matrix being diagonal elements of the Hessian. Since the GP method retains a linear convergence rate, it could be slow when it is approaching the optimal solution. We are interested in finding a method that enjoys a superlinear convergence rate in order to further improve computational efficiency. It should be noted that a superlinear convergence rate is only a local property; only if the iteration point lies within a neighborhood of the optimal solution can we obtain such a rate of convergence. However, we do not know the optimal SUE solution a priori, hence we cannot find an initial point that is near the optimal SUE solution before finishing the computation. Typically, one chooses a logit type route flow pattern based on the free-flow travel costs as the initial point. Therefore, when this initial point is far from the optimal solution, the convergence rate may be slow, and most of the computing time will be consumed in the early and middle stages of the iterative procedure.

In this paper, we propose a modified truncated Newton (MTN) algorithm to solve the path-based SUE problem and investigate the computation efficiency of this algorithm. Traditionally, the way to solve a linear equality constrained problem is as follows. First, we use the variable reduction method to transform this problem into an unconstrained one. Then, we apply the truncated Newton method [33] to solve this reduced unconstrained minimization problem. All the procedures are performed in the reduced variable space. At each iteration point, the next search direction is obtained by approximately solving a reduced Newton equation, using the conjugate gradient method [34]. Evidently, the choice of the reduced variable (nonbasic variable) affects the conditioning of the reduced Newton equation greatly. A poor choice of the reduced variables will significantly deteriorate the convergence rate of the conjugate gradient method. However, we do not know a priori which variables are appropriate to be chosen as the reduced variable. To remedy this drawback, the MTN algorithm is proposed. This algorithm is performed in the original variable space, i.e., both the iteration point and the search direction are in this space. But for each iteration, we approximately solve a reduced Newton equation in the reduced variable space. The search direction is then obtained by the solution result and the null space basis of the constraint matrix. Note that in the MTN algorithm, the reduced variables can be changed from iteration to iteration. This property gives us much more freedom in choosing the reduced variables, and is very useful when solving the path-based SUE problem.

When applying the modified truncated Newton algorithm to the SUE problem, several practical issues are considered in our research, including the computation of the feasible search direction, the determination of the step size, and the principle of the choice of the basic route. These issues are very important to generate a fast and robust solution algorithm for the logit-based SUE problem.

The remainder of this paper is organized as follows. In Section 2, we propose our MTN algorithm for the linearly constrained minimization problem. We also prove the related convergence theorems of the MTN algorithm. In Section 3, we