Applied Mathematical Modelling 39 (2015) 5480–5494

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A one-dimensional inverse problem in composite materials: Regularization and error estimates

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ARTICLE INFO

Article history:
Received 29 November 2012
Received in revised form 1 March 2014
Accepted 5 January 2015
Available online 14 January 2015

Keywords:
Inverse problem
Ill-posedness
Error estimate
Regularization
Composite materials

ABSTRACT

In this paper we investigate an inverse one-dimensional heat conduction problem in multi-layer medium. The inverse problem is first formulated in the frequency domain via Fourier transform technique. An effective regularization method for the stable reconstruction of solution is given with proven error estimates. Several numerical examples are constructed to demonstrate the effectiveness of the proposed method.

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1. Introduction

The transient temperature distribution in a composite medium consisting of several layers in contact has numerous applications in engineering. For instance, it is important in the aerospace engineering to reconstruct the surface temperature and heat flux in high temperature composite materials. One of these works evolved as a boundary identification problem [1] in several-layer domain arises from steel-making industry in which the iron was heated and melted in one container composed of different materials. The result of this work is important to detect any corrosion inside the inner-surface of the container and prevent any disaster damage due to the leakage of the steel fluid. Mathematically, these kinds of problems can be treated as boundary identification problems. Another example is the design problem of shielded thermocouple which is a measurement device used for monitoring the temperature in the hostile environments [2]. The shielded thermocouple device consists of composite materials each with different thermal property. In order to simulate the concrete measurement situations, a mathematical model of the shielded thermocouple is usually formulated as an Inverse Heat Conduction Problem (IHCP) [3] in several-layers domain.

The IHCP arises in thermal manufacturing processes of solids and has attracted much attention [4–7]. In this inverse IHCP problem, it is required to recover the surface temperature and heat flux on an inaccessible boundary from the measurement on an accessible boundary, which is also called as a non-characteristic Cauchy problem of heat equation. It is well known that non-characteristic Cauchy problem of heat equation is ill-posed [8] in the Hadamard sense that any "small" measurement error in the data can induce extremely "large" error in the solution. Under an additional condition, a continuous dependence of the solution on the Cauchy data can be obtained. This is called conditional stability [9]. In other words, in

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http://dx.doi.org/10.1016/j.apm.2015.01.004
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theory we can stably reconstruct the solution of the ill-posed problem under a priori condition which is called the "source condition". Due to the severe ill-posedness of the problem, most classical numerical methods failed to produce satisfactory stable approximation to the solution of the Cauchy problem of heat conduction equation. Some kinds of regularization strategies [10] needs to be employed. For the IHCP in a single-layer domain, theoretical investigation and computational implementation have been well studied, e.g., [11–27]. In alternative to the single-layer IHCP, the multi-layer IHCP is much more difficult in both numerical and theoretical studies. In general, the IHCP in multi-layer domain will be dissolved into an IHCP in each layer and the solution the multi-layer IHCP will be obtained by solving the IHCP in each single layer. This approach is computational inefficient but the author can refer to the recent works [28–30] in which the authors used the method of fundamental solutions combined with regularization to solve each IHCP layer by layer.

This paper aims at establishing a mathematical framework to recover the surface temperature and heat flux on the inaccessible boundary in a two-layer domain [31] as follow.

Consider a two-layer body that consists of the first layer in $0 \leqslant x \leqslant l_1$ and the second layer in $l_1 \leqslant x \leqslant l_2$. The two layers are in perfect thermal contact at $x = l_1$ as displayed in Fig. 1.

Let $k_1, k_2 > 0$ be the thermal conductivities and $\alpha_1, \alpha_2 > 0$ be the thermal diffusivities for the first and second layer, respectively. The temperature distributions in the first and the second layers are denoted by $u_1(x,t)$ and $u_2(x,t)$ respectively. These temperature distributions satisfy the following partial differential equations in the two domains $D_1 := \{x | 0 \leqslant x \leqslant l_1\}$ and $D_2 := \{x | l_1 \leqslant x \leqslant l_2\}$:

$$\frac{\partial u_1}{\partial t} - \alpha_1 \frac{\partial^2 u_1}{\partial x^2} = 0, \quad 0 < x < l_1,\ t > 0, \tag{1.1}$$

$$\frac{\partial u_2}{\partial t} - \alpha_2 \frac{\partial^2 u_2}{\partial x^2} = 0, \quad l_1 < x < l_2,\ t > 0, \tag{1.2}$$

subject to the initial and boundary conditions

$$u_1(x,0) = u_2(x,0) = 0, \quad 0 < x < l_2, \tag{1.3}$$

$$u_2(l_2,t) = g(t), \quad t > 0, \tag{1.4}$$

$$\frac{\partial u_2}{\partial x}(l_2,t) = 0, \quad t > 0, \tag{1.5}$$

$$u_1(l_1,t) = u_2(l_1,t), \quad t > 0, \tag{1.6}$$

$$k_1 \frac{\partial u_1}{\partial x}(l_1,t) = k_2 \frac{\partial u_2}{\partial x}(l_1,t), \quad t > 0. \tag{1.7}$$

For the similar IHCP, Shcheglov [32,33] analyzed the convergence of the problem by using a hyperbolic equation perturbation method. For 2D IHCP in two-layer Cartesian bodies, some numerical approaches have been proposed [34,35]. To the knowledge of the authors, the convergence rate on the IHCP in multi-layer domain has not yet been given.

In this paper, we aim at obtaining an analytical solution to the above IHCP in multi-layer domain via Laplace and Fourier transform techniques. Due to the severe ill-posedness of the problem, a regularization strategy is derived for the stable reconstruction of the solution. For illustration, we present the regularization method based on the analytical solution for the IHCP in a two-layer domain in which we adapt a spectral regularization method and derive the error estimate with proven optimal order. The proposed method is simple and effective [14]. Furthermore, the proposed method can easily be generalized to solve IHCP in the multi-layer domain.

The paper is organized as follows. We first give a theoretical analysis on the IHCP problem in Section 2. In Section 3, the spectral regularization technique is adopted for the reconstruction of stable solution with proven error estimates. Finally, several numerical examples are constructed in the last section to demonstrate the validity of the proposed regularization method although the so-called "inverse crime" [36] has been used.

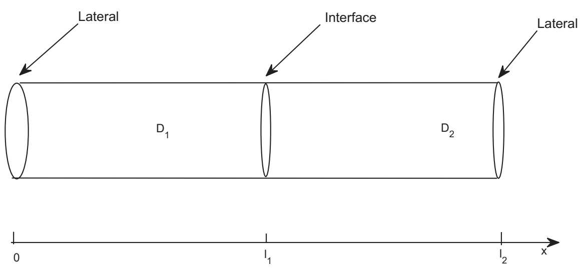

Fig. 1. Schematic illustration of a one-dimensional inverse heat conduction problem in a two-layer domain.

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