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Identification of nonlinear cascade systems with output hysteresis based on the key term separation principle

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ABSTRACT

An approach to modeling and identification of nonlinear cascade systems with a linear dynamic system and an output hysteresis is presented. The proposed mathematical model is based on the application of the key term separation principle and a special form of Coleman–Hodgdon hysteresis model. A least-squares based iterative algorithm with internal variable estimation is used for the cascade systems parameter identification. The feasibility of proposed approach is demonstrated on illustrative examples.

1. Introduction

Hysteresis phenomena are encountered in many different areas of science and technology. The hysteresis is a special type of dynamic nonlinearity, because it is a multi-branching nonlinearity that occurs when the output of a system "lags behind" the input [1,2]. The best known examples of hysteresis affected phenomena can be found in ferromagnetism, piezoelectricity, plasticity, friction, but also in superconductivity, spin glasses, semiconductors, economics, and physiology.

In most cases, the hysteresis is harmful to the accuracy and performance of control systems. For example, actuators and sensors play an important role in the design of control systems, but they display hysteretic behavior in some regime of operation. This dynamic nonlinearity can lead to instability in closed-loop operations, and complicates the task of controller design and analysis [3]. Therefore, it is of great importance to know the hysteresis description, more precisely, to find the best model approximating the systems with hysteretic nonlinearity.

To describe the behavior of hysteretic processes a range of model classes have been proposed and a survey may be found in [4]. For a class of hysteresis systems a first order scalar time-domain differential equation can be used to describe the system behavior [5].

A relatively simple differential model of hysteresis, which is appropriate for the representation of rate independent hysteretic systems, is the so-called Coleman–Hodgdon model proposed in [6–8]. This model (a first-order nonlinear differential equation) is able to capture, in an analytical form, a range of shapes of hysteretic loops, which match the behavior of a wide class of hysteretic systems [9–15].

In this paper, an approach to modeling and identification of cascade systems with an output hysteresis using the Coleman–Hodgdon model is presented. The proposed mathematical model is based on the application of the key term separation principle and a special form of Coleman–Hodgdon hysteresis model [16]. This cascade model is linear in parameters

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and enables to perform the identification of cascade systems with an output hysteresis on the basis of available input/output data. For the model parameter estimation a least-squares based iterative algorithm with internal variable estimation is proposed. The feasibility of proposed approach is demonstrated on illustrative examples.

2. Coleman–Hodgdon hysteresis model

The differential model of hysteresis according to [6–8] is a representation of a rate-independent dynamic effect in the form of the first order nonlinear differential equation in the time domain

$$\dot{y} = f(x,y)|\dot{x}| + g(x,y)\dot{x}, \tag{1}$$

where y is the output and x is the input. Both y and x are real-valued functions of time with piecewise continuous derivatives, $\dot{y}$ and $\dot{x}$. In the following, the differential model of hysteresis based on a function $f(x,y)$ that is affine in y, and a function $g(x,y)$ that is constant in y, will be considered. Then the hysteresis model can be written as:

$$\dot{y} = \alpha[f(x) - y]|\dot{x}| + g(x)\dot{x}, \tag{2}$$

where $0 < a$ is a real number, $f(.)$ is odd, monotone increasing and piecewise continuously differentiable real-valued function, $g(.)$ is even, piecewise continuous real-valued function.

The above-described Coleman–Hodgdon hysteresis model can be written in the discrete form as

$$y(t+1) - y(t) = \alpha\,\{f[x(t)] - y(t)\}|x(t+1) - x(t)| + g[x(t)][x(t+1) - x(t)], \tag{3}$$

where $t = 1,2,3,\ldots$, and can be used for a broad class of hysteretic systems by an appropriate choice of the functions $f(.)$ and $g(.)$. In the simplest case [9], the following functions can be chosen:

$$f[x(t)] = \begin{cases} m_1 D_1, & \text{if} \quad D_1 < x(t), \\ m_1 x(t), & \text{if} \quad -D_1 \leqslant x(t) \leqslant D_1, \\ -m_1 D_1, & \text{if} \quad x(t) < -D_1, \end{cases} \tag{4}$$

$$g[x(t)] = \begin{cases} 0, & \text{if} \quad D_1 < x(t), \\ b, & \text{if} \quad -D_1 \leqslant x(t) \leqslant D_1, \\ 0, & \text{if} \quad x(t) < -D_1, \end{cases} \tag{5}$$

where m_1 is the slope of the central segment of the function $f(.)$, the constant $D_1 > 0$ determines the range of the central segment as well as the range of $g(.)$ with the constant value b – see Fig. 1. An example of the major hysteresis loop and some minor loops generated by the hysteresis model (3) based on the functions (4) and (5) using triangular inputs $x(t)$ with different amplitudes is shown in Fig. 2.

In this case, the hysteresis model is characterized by 4 parameters: a, m_1, D_1, and b. However, to estimate the model parameters on the basis of inputs $x(t)$ and outputs $y(t)$ is not easy because of the complicated mathematical model description given by (3)–(5).

A way how to simplify this model description is to consider appropriate analytical forms of (4) and (5) enabling to solve the problem of hysteresis model parameter estimation as a pseudo-linear one [16]. Using the switching function defined as

$$h(s) = \begin{cases} 0, & \text{if } s > 0, \\ 1, & \text{if } s \leqslant 0, \end{cases} \tag{6}$$

the functions (4) and (5) can be rewritten as follows:

$$f[x(t)] = m_1\, x(t)\{1 - h[D_1 - |x(t)|]\} + m_1\, D_1\, h[D_1 - |x(t)|]sign[x(t)], \tag{7}$$

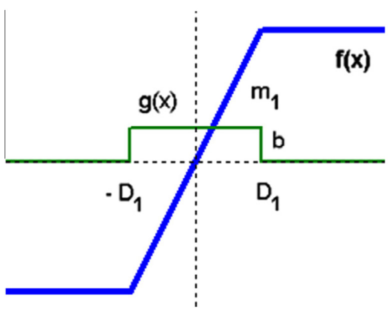

Fig. 1. Functions f and g.