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Modified reproducing kernel method for singularly perturbed boundary value problems with a delay

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ABSTRACT

This paper is devoted to the numerical treatment of a class of singularly perturbed delay boundary value problems with a left layer. The method is proposed based on the reproducing kernel theory and the error estimate of the present method is established. A numerical example is provided to show the effectiveness of the present method. Numerical results show that the present method is accurate and efficient.

1. Introduction

Singularly perturbed delay differential equations arise frequently in many scientific and technical fields. They can be applied to neurobiology [1], optimal control theory [2] and models for physiological processes [3]. Recently, there has been growing interest in developing approximate methods for singularly perturbed problems with delays. However, the presence of boundary layers and delayed terms makes it difficult to develop valid numerical methods for such problems.

Recently, considerable attention has been towards to numerical solutions of singularly perturbed delay differential equations. Some new techniques have appeared in the literature[4–19]. Amiraliyev, Erdogan, Amiraliyeva and Cimen [4–7] proposed exponentially fitted methods for singularly perturbed delay initial and boundary value problems. Kadalbajoo, Sharma and Gupta [8–10] presented some methods for solving singularly perturbed delay boundary value problems. Rai and Sharma [11–13] developed numerical methods for singularly perturbed delay differential turning point problems. Patidar and Sharma [14] gave uniformly convergent non-standard finite difference methods for singularly perturbed differential–difference equations with delay and advance. Mohapatra and Natesan [15] proposed a numerical method for a class of singularly perturbed differential–difference equations with small delay and shift terms, based on the upwind finite difference operator on an adaptive grid. Subburayan and Ramanujam [16,17] presented initial value techniques for singularly perturbed convection–diffusion problems with a delay. Chakravarthy and Rao [18,19] developed the modified Numerov and finite difference methods for singularly perturbed differential–difference equations.

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Recently, based on reproducing kernel theory, the reproducing kernel method (RKM) has been proposed and applied to integral equations, local and nonlocal boundary value problems [20–34]. The reproducing kernel particle method has been presented and developed in [35–38]. Both methods are meshfree methods and based on the good properties of reproducing kernel. The RKM can avoid solving the reduced systems of algebraic equations. However, they fails to solve singularly perturbed delay boundary value problems.

Motivated by the work of [7], we consider the RKM for following singularly perturbed problems:

$$\begin{cases} \varepsilon u''(x) + a(x)u'(x) + b(x)u(x-r) = f(x), & x \in \Omega = (0,l), \\ u(x) = \Phi_0(x), \quad x \in \Omega_0 = [-r, 0], & u(l) = B, \end{cases} \tag{1.1}$$

where $0 < \varepsilon \ll 1, a(x) \geqslant \alpha > 0, a(x), b(x), \Phi_0(x)$ and $f(x)$ are assumed to be sufficiently smooth functions satisfying certain regularity conditions, r is a constant delay and $l < 2r$.

From [7], (1.1) exhibits a boundary layer near $x = 0$.

2. Method

In this section, we will develop a modified RKM for (1.1).

Introduce a new unknown function

$$v(x) = u(x) - \lambda(x), \tag{2.1}$$

where

$$\lambda(x) = \Phi_0(0)\left(1 - \frac{x}{l}\right) + \frac{Bx}{l}.$$

Problem (1.1) is reduced to

$$\begin{cases} \varepsilon v''(x) + a(x)v'(x) + b(x)v(x-r) = g(x), & x \in \Omega = (0,l), \\ v(x) = \Phi(x), \quad x \in \Omega_0 = [-r, 0], & v(l) = 0, \end{cases} \tag{2.2}$$

where

$$\Phi(x) = \Phi_0(x) - \lambda(x), \quad g(x) = f(x) - [a(x)\lambda'(x) + b(x)\lambda(x-r)].$$

Let

$$Lv(x) = \begin{cases} \varepsilon v''(x) + a(x)v'(x), & x \in (0, r], \\ \varepsilon v''(x) + a(x)v'(x) + b(x)v(x-r), & x \in (r, l]. \end{cases}$$

Problem (2.2) can be equivalently reduced to the problem of finding a function $v(x)$ satisfying

$$\begin{cases} Lv(x) = F(x), \\ v(0) = \Phi(0) = 0, \quad v(l) = 0, \end{cases} \tag{2.3}$$

where

$$F(x) = \begin{cases} g(x) - b(x)\Phi(x-r), & x \in (0, r], \\ g(x), & x \in (r, l]. \end{cases}$$

We will give the approximate solution of (2.3) in the following reproducing kernel space $W^4[0,l]$.

Definition 2.1. $W^4[0,l] = \{u(x)|u'''(x)$ is absolutely continuous, $u^{(4)}(x) \in L^2[0,l], u(l) = 0\}$. The inner product and norm in $W^4[0,l]$ are given, respectively, by

$$(u(y), v(y))_4 = u(0)v(0) + u'(0)v'(0) + u''(0)v''(0) + u'''(0)v'''(0) + \int_0^l u^{(4)}v^{(4)}dy,$$

and

$$\|u\|_4 = \sqrt{(u,u)_4}, \quad u, v \in W^4[0,l].$$

From [20,21], it is easy to prove $W^4[0,l]$ is a reproducing kernel space and obtain its reproducing kernel

$$k(x,y) = \begin{cases} k_1(x,y), & y \leqslant x, \\ k_1(y,x), & y > x, \end{cases} \tag{2.4}$$