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An efficient collocation algorithm for multidimensional wave type equations with nonlocal conservation conditions

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ABSTRACT

In this paper, we derive and analyze an efficient spectral collocation algorithm to solve numerically some wave equations subject to initial-boundary nonlocal conservation conditions in one and two space dimensions. The Legendre pseudospectral approximation is investigated for spatial approximation of the wave equations. The Legendre–Gauss–Lobatto quadrature rule is established to treat the nonlocal conservation conditions, and then the problem with its nonlocal conservation conditions are reduced to a system of ODEs in time. As a theoretical result, we study the convergence of the solution for the one-dimensional case. In addition, the proposed method is extended successfully to the two-dimensional case. Several numerical examples with comparisons are given. The computational results indicate that the proposed method is more accurate than finite difference method, the method of lines and spline collocation approach.

1. Introduction

For several decades, spectral method has been developed to obtain more accurate solutions for differential equations, spectral method (see for instance [1–5]) is one of the family of weighted residual methods for solving various problems, including nonlinear differential equations [6–9], integral equations [10,11], integro-differential equations [12,13], fractional orders differential equations [14–17].

The pseudospectral method is well-known for its high accuracy and has been applied extensively in scientific computation, see [1,18–21] and the references therein. Doha et al. [22] constructed the Jacobi–Gauss–Lobatto pseudospectral schemes for numerically solving 1 + 1 nonlinear Schrodinger equations. In the same line of thought, a Chebyshev–Gauss–Radau collocation method in combination with the implicit Runge–Kutta scheme have been investigated by Doha et al. in [23] to obtain more accurate numerical solutions for hyperbolic systems of first order. Meanwhile, Doha et al. [24] proposed the Jacobi–Gauss–Lobatto pseudospectral scheme for solving nonlinear coupled hyperbolic Klein–Gordon. Moreover, for time-dependent coefficients problem, Bhrawy [25] presented and applied an efficient numerical algorithm based on Jacobi–Gauss–Lobatto pseudo-spectral scheme for solving complex generalized Zakharov system subject to Dirichlet bound-

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ary conditions. It is well-known that the pseudospectral method achieves the exponential convergence for the spatial approximation of initial-boundary value problems.

Numerical solutions of the wave equations subject nonlocal boundary conditions continues to be a major research area with widespread applications in modern physics and technology [26–31]. Wave equation [32–34] is one of the hyperbolic PDEs [35–37]. The authors of [38] applied the variational iteration scheme for the one-dimensional wave equation with mixed and integral conditions. Also, various numerical solution for wave equation subject to nonlocal conservation condition have been studied in [39–44]. For numerical solutions of ODEs with integral boundary conditions like forced Duffing equations, see the articles [45,46]. Moreover, fractional differential equations based on integral boundary conditions have been studied in many articles, see for examples [47–52].

The main destination of this paper is to extend the application of the Legendre–Gauss–Lobatto collocation (L–GL–C) approximation to solve wave type equations with initial-boundary nonlocal conservation conditions. We approximate the solution of such problem for space variables using a Legendre polynomials. The approximate solution is then evaluated at the Legendre Gauss–Lobatto (L–GL) interpolation nodes. In addition, the spatial partial derivatives are approximated at these nodes. Thereby, the problem is exactly satisfied at $N-1$ interior points of L–GL interpolation nodes. Moreover, the nonlocal conservation conditions are efficiently treated by L–GL quadrature rule at $(N+1)$ nodes. A proper initial value software can be applied to solve the resulted system of ODEs. The convergence of the solution of L–GL–C method is analyzed. Moreover, this algorithm is developed to solve the two-dimensional wave equation. Finally, the powerful and effectiveness of the method are demonstrated by solving several problems.

A brief outline of this paper is as follows. Section 2 summarizes some properties of Legendre polynomials needed for rest the paper. In Section 3, the L–GL–C technique is presented for the one- and two-dimensional wave equations, and the convergence of the solution of L–GL–C method. Section 4 presents several numerical examples for wave equations subject to nonlocal conservation conditions. Finally, some concluding remarks are given in Section 5.

2. Legendre polynomials and L–GL interpolation

In this section, we recall some results on the L–GL interpolation, which play important roles in the proposed collocation scheme. The Legendre polynomials $L_k(x)$ $(k=0,1\ldots)$ satisfy the following Rodrigue's formula

$$L_k(x)=\frac{(-1)^k}{2^k k!}D^k((1-x^2)^k), \tag{2.1}$$

also we recall that $L_k(x)$ is a polynomial of degree k and therefore $L_k^{(q)}(x)$ (the qth derivative of $L_k(x)$) is given by

$$L_k^{(q)}(x)=\sum_{i=0(k+i=even)}^{k-q} C_q(k,i)L_i(x), \tag{2.2}$$

where

$$C_q(k,i)=\frac{2^{q-1}(2i+1)\Gamma\left(\frac{q+k-i}{2}\right)\Gamma\left(\frac{q+k+i+1}{2}\right)}{\Gamma(q)\Gamma\left(\frac{2-q+k-i}{2}\right)\Gamma\left(\frac{3-q+k+i}{2}\right)}.$$

Next, denote by (u,v) and $||u||$ the inner product and the norm of space $L^2(-1,1)$. The set of $L_k(x)$ is a complete orthogonal system in $L^2(-1,1)$, namely

$$\left(L_k(x),L_j(x)\right)=\int_{-1}^{1}L_k(x)L_j(x)\,dx=h_k\delta_{jk}, \tag{2.3}$$

where $h_i=\frac{2}{2i+1}$. Thus for any $u\in L^2(-1,1)$,

$$u(x)=\sum_{i=0}^{\infty}a_iL_i(x),\quad a_i=\frac{1}{h_i}\int_{-1}^{1}u(x)L_i(x)\,dx. \tag{2.4}$$

For any positive integer N, let $S_N(-1,1)$ be the set of all polynomials of degree at most N. Due to the L–GL quadrature, it follows for any $\phi\in S_{2N-1}(-1,1)$ that

$$\int_{-1}^{1}w(x)\phi(x)dx=\sum_{j=0}^{N}\varpi_{N,j}\phi(x_{N,j}), \tag{2.5}$$

where $x_{N,j}$ $(0\leqslant j\leqslant N)$ and $\varpi_{N,j}$ $(0\leqslant j\leqslant N)$ are the nodes and Christoffel numbers of L–GL interpolation on the interval $[-1,1]$, respectively. Now, it is quite useful in the sequel to introduce the following discrete inner product and norm

$$(u,v)_N=\sum_{j=0}^{N}u(x_{N,j})\,v(x_{N,j})\,\varpi_{N,j},\qquad ||u||_N=(u,v)_N^{\frac{1}{2}}. \tag{2.6}$$