Applied Mathematical Modelling 39 (2015) 5659–5680

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A refined laminated plate theory accounting for the third-order shear deformation and interlaminar transverse stress continuity

X. Wang, G. Shi *

Department of Mechanics, Tianjin University, Tianjin 300072, China

ARTICLE INFO

Article history:
Received 3 September 2013
Received in revised form 28 December 2014
Accepted 14 January 2015
Available online 27 January 2015

Keywords:
Laminated composite plate
Shear deformable plate theory
Transverse shear function
Interlaminar stress
Continuity conditions

ABSTRACT

A variational consistent third-order shear laminated plate theory accounting for the transverse shear stress continuity at the interlaminar interfaces of laminated plates is developed in this paper. The transverse shear function used in this new laminated plate theory is based on the kinematics in the third-order shear deformation plate theory proposed by Shi (2007). The variational principle is employed to derive the variational consistent equilibrium equations in terms of displacements and the variational boundary conditions in terms of displacements and equivalent stress resultants. The continuity conditions of the in-plane displacements and transverse shear stresses at the interlaminar interfaces of laminated plates are enforced by the Heaviside step functions and continuity coefficients. The resulting new laminated composite plate theory accounting for interlaminar continuity has only five independent field variables. Furthermore, the number of the field variables in the present third-order shear laminated plate theory is the same as that used in the first-order shear deformation plate theory. The refined laminated plate theory is applied to solve the bending problems of four laminated composite plates with different lamination schemes and different aspect ratios to evaluate its reliability and accuracy. The resulting analytical solutions of both deflections and stresses agree well with the 3D elasticity solutions and the numerical results of finite element analysis. The result comparison with other laminated plate theories shows that new laminated plate theory accounting for the interlaminar continuity proposed in this paper yields more accurate displacements and stresses than other laminated plate theories with five global variables. Because only five field variables are used in this new laminated plate theory with interlaminar continuity, this refined laminated plate theory can be used as an accurate and efficient theoretical model for the finite element analysis of laminated composite plates.

1. Introduction

Composite materials are finding an increasing uses in various industries, such as aircraft and aerospace industry, automotive industry, marine and offshore industry, construction, consumer products, sport goods etc. [1–3]. The most attractive mechanical properties of composite materials are the high specific modulus and high specific strength. Composite materials also possess high fatigue strength, ease of formability, wide range of operating temperatures (thermoplastic resins), low (even negative) coefficient of thermal expansion, high damping, high corrosion resistance. Because of the inhomogeneous

* Corresponding author.
E-mail address: shi_guangyu@163.com (G. Shi).

http://dx.doi.org/10.1016/j.apm.2015.01.030
0307-904X/

and anisotropic nature, the mechanical properties of the composite materials are much more complicated than the conventional materials. Therefore, the significant increase in the use of composite materials have attracted many researchers for the development of rigorous mathematical methods capable of modeling, designing and optimizing of the composite under any given set of conditions the past four decades [4,5].

Generally speaking, the laminated plate theories can be broadly grouped into the following six categories: classical lamination theory; the first-order shear deformation theory; higher-order shear deformation theories; layerwise lamination theory; global–local higher-order plate theory; and continuum-based 3D elasticity theory.

The classical plate theory and first-order shear deformation plate [6,7] cannot accurately describe the mechanical behavior of laminated composite plates especially the transverse shear stress at the interfaces. The various higher-order shear deformation theories [4,8–17] are widely used in engineering field. However, because the higher-order plate theories cannot take the continuity conditions of displacement and shear stresses into consideration, the accuracy of analytical solutions cannot meet the requirements in engineering applications. The layerwise theory proposed by Carrera et al. [18–21] is very popular in recent years, because the theory can satisfy the continuity conditions and give more accurate results. The layerwise theory has its own shortcoming, which is the unknowns used in the plate theories are dependent on the number of the laminae, leading to the extensive computation of analysis. Carrera proposed a theoretical model named as Carrera's Unified Formulation (CUF) [22]. This model is capable of obtaining accurate results of displacement and stresses, and is valid for all kinds of engineering structures from plates to double-curved shells. The continuum-based 3D elasticity theory proposed by Pagano [23–25] is very accurate. However, it is only applicable in the analysis of laminated plates with simple geometries and boundary conditions but not appropriate for the large scale numerical analysis of laminated plates. The global–local higher-order plate theories [26,27] with independent variables can give accurate results. However, there are more global field variables used in these theories. For example, 13 global field variables are employed in the global–local plate theories presented by Wu and Chen [27].

There are some new modifications and developments on the composite plate and shell theories in recent years. Tornabene et al. [28,29] presented a general formulation of 2D higher-order equivalent single layer theory. The theoretical framework covers the static and dynamic analysis of shell structures by using a general displacement field based on Carrera's Unified Formulation (CUF) [22]. Tornabene et al. [28,29] made an extension of the CUF to the doubly-curved shells structures by employing the Generalized Differential Quadrature and Generalized Integral Quadrature techniques. The results are in agreement with the solutions given by 3D finite element method. Moreover, a three dimensional stress recovery procedure [29] based on the shell equilibrium equations is used to calculate through-the-thickness quantities. Nedri et al. [30] presented a new refined hyperbolic shear deformation theory based on the assumption that the transverse displacements consist of bending and shear components where the bending components do not contribute to shear forces, and likewise, the shear components do not contribute to bending moments. The number of independent unknowns in the theory is four, as against five variables in other shear deformation theories. And the theory can give accurate results in dynamic analysis.

However, the higher-order plate theories based on the simple polynomials are the most widely used in engineering applications. The variational consistent third-order shear deformation plate theory given by Shi [15] is not only simple but also has some special features, such as the averaged rotations used for the field variables and the capability to correctly characterize boundary layer effects [31]. The third-order plate theories proposed by Shi can give more accurate results in static and dynamic analyses of anisotropic plates than other plate theories [31–33], which will be expounded in detail in next section. By using the kinematics in Shi's plate theory [15], Wang and Shi [34] presented a sandwich plate theory where the effect of transverse normal strain is taken into consideration with only five independent field variables. This equivalent single layer sandwich plate theory is not only simple, but also capable of achieving the accuracy of layerwise sandwich plate theories. Furthermore, Shi's plate theory is very suitable for the finite element analysis of laminated plates as only five global generalized displacement variables are used in Shi's plate theory. However, the third-order shear deformation theory presented by Shi was proposed for single-layer anisotropic plates, it cannot satisfy the interlaminar continuity conditions of transverse shear stresses in laminated plates.

This paper has two objectives, one is to present a new laminated plate theory with five global field variables but accounting for the interlaminar stress continuity conditions, and the other is to evaluate the accuracy of the interfacial transverse shear stress predicted by the new laminated plate theory through a comprehensive investigation with both analytical and numerical results. The comparison study conducted in this paper demonstrates that the new laminated plate theory accounting for the interlaminar continuity proposed in this paper yields almost identical results to the solutions given by the 3D-elasticity when the aspect ratios of the laminated plates $a/h \geqslant 20$, which are the aspect ratios of the practical laminated plates in various engineering applications. And the present laminate plate theory yields more accurate results of both the displacements and transverse shear stresses of the laminated plates with all the aspect ratios than other laminated plate theories. Only the theoretical derivation and the analytical solutions of the new laminated plate theory are presented in this paper. Because only five field variables are used in the new laminated plate theory although the interlaminar continuity of transverse shear stresses is taken into account, the present refined laminated plate theory can be used as an accurate and efficient theoretical model for the finite element analysis of laminated composite plates.