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Empirical formulation of stress concentration factor around an arbitrary-sized spherical dual-cavity system and its application to aluminum die castings

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ABSTRACT

An empirical formula for the stress concentration factor is developed for an unequal-sized cavity pair in an arbitrary orientation. Three-dimensional finite element linear elastic analyses are performed to evaluate the stress concentration factors for different sizes, orientations, and separations of cavities. A suitable mathematical function is chosen to fit the numerical results of the finite element analyses. An application is given for evaluating the maximum stress concentration factor, which governs fatigue crack initiation in aluminum die cast test pieces from an engine block. From the X-ray CT image, the location and geometry of the gas pores are evaluated so as to develop the proposed empirical formula for this actual multi-pore system simplified to a dual spherical pore system. A proof of the formula is shown by comparison with voxel finite element analysis. The proposed empirical formula can be satisfactorily used as a scientific guideline for selecting a casting method for car engine blocks from a fatigue crack initiation perspective.

1. Introduction

Aluminum alloys are extensively used in automobile parts because of their high specific strength and moldability. These engineering materials contain some defects due to different manufacturing processes. One such manufacturing defect inherent to high-pressure die cast aluminum alloy is micro-porosity. Most of the components used in the automobile industry are made using the high-pressure die casting process, which introduces micro-cavities, clustered pores, cold flakes, etc., into the finished products. These cavities often act as sources of high stress concentrations. The fatigue lives of these components are strongly affected by the presence of such high stress concentration regions. So, in order to assess the integrity and fatigue life of these components, it is important to have a thorough knowledge of the stress concentration arising due to the presence of cavities. The geometrical properties of the cavities, such as their locations, separation distance, and shape, will affect the stress concentration and hence the fatigue life of the component. Therefore, a systematic study of

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the geometrical parameters of these micro-cavities on the fatigue life is important to the understanding of fatigue crack initiation in aluminum die castings.

Many experimental studies have shown that the presence of gas pores has the most critical influence on the fatigue strength of die cast aluminum alloys since they act as preferential sites for fatigue crack initiation [1,2]. It is believed that a region of high stress concentration is created around certain gas pores, which accelerates the fatigue crack initiation process and reduces the fatigue lives of such components. Some researchers have used X-ray computed tomography (CT) to visualize and track the fatigue crack initiating gas pores [3] and to quantify the stress concentrations around those pores by image-based finite element methods [4,5]. While it is possible to identify which gas pore has caused fatigue crack initiation by comparing X-ray CT images from before and after the fatigue tests, it is not possible to predict the vulnerability of such gas pores without conducting fatigue tests. Some researchers have reported surface pores to be more critical [6], while in some cases, cracks have initiated from bigger gas pores in the interior and also from small gas pores at random locations. It has been reported that the fatigue lives of such specimens vary by as much as four orders of magnitude for the same nominal stress and similar average porosity. In order to effectively predict such variability, the interaction between the gas pores should be considered. It is not well understood which individual gas pore is critical for fatigue crack initiation.

Since stress concentration is an important factor in fatigue crack initiation, a detailed study on the effect of pore-to-pore interactions on the stress concentration is required. In actual components, such as automobile engines, the cavities are numerous and randomly distributed. If some methodology can be developed to conveniently predict stress concentrations in such a multi-pore system (which is representative of an actual system), then it will have much practical significance. An analytical closed-form solution for such a system is not available, although some numerical methods have been proposed in past research. Chiang used the Eshelby tensor and equivalent inclusion principle to describe the stress concentration around an ellipsoidal cavity [7,8]. Sternberg and Sadowsky proposed using Papcovich–Boussinesq displacement functions to solve the stress distribution around a pair of equal-sized cavities in an infinite continuum [9]. Miyamoto et al. [10] used Papkovich–Neuber stress functions in spherical coordinates to determine the stress distribution around spherical cavities. Eubank [11] developed and explicit series solution for equal size spherical cavities subjected to torsion. Tsuchida [12,13] extended Miyamoto's approach to determine the stress interference arising from more than two equal-sized spherical cavities. Sadraie et al. [14] proposed a boundary spectral method to solve the stress distribution around an array of an arbitrary number of equal-sized spherical cavities. The numerical methods used in these studies often employ complex mathematical functions that are solved using infinite series and demand more computation time. The accuracy of the result is also affected by number of terms in truncated infinite series, and there are also difficulties in achieving convergence in many situations. Some researchers have also proposed realistic finite element modeling using multiphase elements for 3-D microstructure [15,16]. Sukumar et al. [17] has employed level set techniques in extended finite element method for holes and inclusions. Image based finite element method could also be carried out from the X-ray CT image data. However, such model will be time consuming and image data would be huge enough to demand heavy computational cost. It is impractical to build finite element mesh from X-ray CT of an entire die cast part and carry out subsequent image based finite element analysis. Considering the fact that the dimensions of gas pores are in order of few tens of micrometer while that of die cast part in order of few tens of centimeters, the finite element mesh would have to be fine enough to capture the geometry of these gas pores. This would generate huge number of nodes and elements which are impractical to handle. This motivated us to explore development of an empirical formula which would require only geometrical information about pore location and size and would able to quickly calculate the stress concentration factors with reasonable accuracy and with less computational effort. So the industrial personnel can use such formula to quickly decide whether to accept or reject a particular die casted part in a mass production process. Finite element method would prohibit such practical application.

In a recent study by Bidhar et al. [18], an empirical formula was developed with which the severity of the highly stressed region can be predicted based on inter-cavity separation and cavity orientation with respect to the loading direction for equal-sized cavities. In this paper, this has been extended to a more general case of unequal-sized cavity pairs in order to more closely represent the actual gas pores found in aluminum die castings, where the gas pores are of different sizes. The stress concentration from unequal-sized cavities is investigated to understand what geometrical property is critical for the stress concentration. The empirical formula for an equal-sized cavity pair is modified to incorporate various sizes of cavity pairs separated at different inter-cavity distances and having different orientations with respect to the loading direction. Finite element analyses are made to determine the stress concentration factors in each case, which are then used to construct a mathematical formula using non-linear surface fitting techniques. The validity of the empirical formulation is demonstrated for a real engine block specimen. Lastly, a comparative study is performed with image-based finite element analysis.

2. Stress concentration in a dual-cavity system

The fatigue performance of aluminum pressure die castings is greatly influenced by the presence of gas pores. These gas pores are sources of discontinuity, and their boundaries often act as stress concentrators, which facilitate fatigue crack initiation. Usually, these voids are formed during the manufacturing process and are of irregular shapes and sizes. Evaluating the stress field around such irregular boundaries is difficult, and a closed-form solution is not available for such geometries. For simplicity, such voids are assumed to be regular spherical cavities. So, it is interesting to study the interference of stress