Applied Mathematical Modelling 39 (2015) 5033–5046

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Transient acoustic radiation from an eccentric sphere

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ARTICLE INFO

Article history:
Received 17 March 2013
Received in revised form 8 March 2015
Accepted 13 April 2015
Available online 24 April 2015

Keywords:
Exact solution
Non-stationary fluid–structure interaction
Spherical shell
Non-uniform thickness

ABSTRACT

A vigorous semi-analytical model is offered to describe the 3D coupled nonaxisymmetric non-steady acousto-elastodynamic behavior of a submerged solid sphere with an off-center fluid-filled spherical cavity, acted upon by general distributed time-varying mechanical loads at its internal and/or external boundaries. The solid medium is designated by the 3D Navier's linear elasticity model, while the internal/external ideal compressible fluids are assumed to obey the classical linear acoustic theory. Laplace transformation is applied to the time variable, and the separation of variables technique together with the pertinent fluid/solid interface conditions, the classical orthogonality properties of spherical harmonics, and a modified form of the translational addition theorem for spherical vector wave functions, are utilized to attain the final matrix equations in terms of unknown modal coefficients. Durbin's Laplace transform inversion algorithm is subsequently applied to compute the pressure/displacement time response histories of water-submerged eccentric and coaxial metallic spheres excited by a couple of external concentrated Ricker-pulse radial excitations. Also, some key characteristics of the transient structure/liquid interaction phenomena with respect to cavity eccentricity are noted based on selected two-dimensional visualizations of the internal/external sound fields. Lastly, the accuracy of numerical simulations is verified by employing a standard FEM software.

1. Introduction

Numerous researchers have investigated the transient structure/fluid interaction of submerged elastic cylindrical/spherical shell structures experiencing impulsive acoustical or mechanical excitations [1–3]. For example, Lou and Klosner [4] used modal expansion and Laplace transform methods to study transient response of a submerged spherical shell to impulsive loads. Akkas and Engin [5] employed the residual potential technique to investigate the transient acoustic response of a spherical shell to a Heaviside point load. Zhang and Zhang [6] numerically simulated the acoustic energy streamlines of an internally-excited submerged spherical shell. Stepanishen [7] utilized *in-vacuo* modal vector expansions to calculate the transient vibratory responses of fluid-submerged flexible shell structures of revolution driven by wideband axisymmetric transient excitations. Jones-Oliveira and Harten [8] also used the *in-vacuo* eigenfunction expansions to study axisymmetric fluid–solid interaction of a submerged thin spherical shell under an explosive shock wave. Zakout [9] investigated the transient modal response of a surrounding acoustic fluid due to a step axisymmetric stress acting on the inner surface of a

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http://dx.doi.org/10.1016/j.apm.2015.04.010
0307-904X/

submerged spherical shell. Bhattacharyya and Premkumar [10] formulated non-reverberating surface conditions in spheroidal coordinates for non-stationary structure/fluid coupling analysis of immersed spheroidal bodies including that of a spherical shell. Chappell et al. [11] used a coupled time domain finite and boundary element technique to characterize the non-stationary sound radiation by a thin elastic spherical shell. Lee et al. [12] employed a weighted combination of the delayed and progressive potentials for external acoustic interaction analysis of submerged thin flexible spherical shells. Geers and Sprague [13] formulated a spherical computational boundary, based on the residual-potential method, for transient acoustic analysis of a step-wave-excited elastic spherical shell.

When geometric imperfections exist in nominally axisymmetric components, the proportionality is lost, separation of the degraded resonant frequencies may happen, and the performance of the element could be severely debilitated [14]. A number of scientists have exploited the classical (translational) addition theorems [15,16] to examine time-harmonic elastic wave interactions in eccentric domains [17,18]. For example, Hasheminejad et al. [18] employed the Helmholtz equations for the internal/external acoustic domains, the Navier's equations of elastodynamics for the solid material, along with the technique of separation of variables and the addition theorems for spherical vector wave functions [16], to develop a rigorous exact analytic (normal mode series) solution for non-axisymmetric *steady-state* acoustic radiation from a doubly-fluid-loaded eccentric hollow elastic sphere excited by arbitrary time-harmonic (internal and/or external) surface loads. The numerical results exposed the key effects of cavity eccentricity, load distribution, and driving frequency on sound radiation characteristics of the submerged structure. In particular, the far-field pressure directionality patterns, the mechanical admittance spectra, coupling between the dominant resonant modes, and diffraction of circumnavigating shell-borne peripheral waves, were discussed. Subsequently, Hasheminejad and Mousavi-Akbarzadeh [1] assumed a set of equally spaced virtual sources in the axial direction to extend their former work on steady-state acoustic radiation by a fluid-immersed eccentric elastic cylinder [17] to the transient case. Semi-analytical solutions were obtained in the Laplace domain in form of discrete summations of simple two dimensional solutions with different axial wave numbers. Time domain solutions for non-stationary sound radiation from the eccentric cylinder were then calculated by direct application of Durbin's numerical inverse Laplace transform scheme.

In the current paper, we shall rather follow the general procedure presented in Ref. [1], and utilize the 3D time-domain linear acoustic wave equation, the Navier's equations of transient elastodynamics, the separation of parameters in spherical coordinates, in conjunction with a modified form of the addition theorem for spherical vector wave functions, in order to extend our above-mentioned work on steady-state acoustic radiation from an eccentric elastic sphere [18] to the transient situation. Ultimately, a rigorous semi-analytical coupled acousto-elastodynamic time-domain solution aimed at the 3D non-axisymmetric *non-steady* acoustic radiation from the submerged fluid-filled eccentric sphere under general distributed time-varying surface excitations is presented. Such analysis is of both academic and practical concern because its essential worth as a standard problem in engineering acoustics. The presented set of converged time-domain solutions can particularly supplement the exploratory subaquatic testing techniques for object categorization and nondestructive characterization of eccentric parts of spherical geometry [19–21]. It may further be used for confirmation of solutions obtained by stringently numerical and/or approximate asymptotic techniques.

2. Formulation

2.1. Basic field equations

Consider a hollow eccentric elastic isotropic sphere of outer radius b, with "e" denoting its eccentricity, (ρ, μ, λ) referring to its mass density and elastic moduli, respectively, filled (in the internal cavity of radius a) with a perfect acoustic liquid of density ρ_{in} and sound velocity c_{in}, immersed in another acoustic liquid of density ρ_{ex} and sound velocity c_{ex}, and subjected to transient arbitrary distributed normal, transverse, or axial general forces at its inner $(F^{in}_{r,\theta,\phi})$ and/or outer $(F^{ex}_{r,\theta,\phi})$ boundaries, as depicted in Fig. 1 (only the external radial load is shown, and the internal forces are not exposed to increase clarity). In the absence of body loads, the elastic displacement field complies with the standard Navier equation of motion [22] subjected to the pertinent surface conditions:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{u}), \tag{1}$$

where $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2}$ is Laplacian, and the material movement vector can be advantageously broken down into the volumetric and solenoidal elements in the form [14,23]:

$$\mathbf{u} = \mathbf{L} + (\mathbf{M} + \mathbf{N}) = \boldsymbol{\nabla}\boldsymbol{\varphi} + [\boldsymbol{\nabla} \times (\mathbf{e}_r r \psi) + \ell \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times (\mathbf{e}_r r \chi)], \tag{2}$$

where ℓ = b is introduced so that the dimension of both terms are the same (note that the above equation can readily be obtained from Eq. (8.2.11) in p. 720 of Ref. [23] in conjunction with definition "v" in page 723 of the same reference). Also, the material movement potentials (φ, ψ, χ) satisfy the following wave equations (e.g., see Eqs. (8.2.8) and (8.2.17) in pp. 719–720 of Ref. [23]):

$$c_L^2 \nabla^2 \varphi = \ddot{\varphi}, \quad c_S^2 \nabla^2 \psi = \ddot{\psi}, \quad c_S^2 \nabla^2 \chi = \ddot{\chi}, \tag{3}$$