Applied Mathematical Modelling 39 (2015) 5073–5090

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Vibration response of laminated composite plate having weakly bonded layers

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A R T I C L E I N F O

Article history:
Received 24 November 2013
Received in revised form 15 March 2015
Accepted 10 April 2015
Available online 22 April 2015

Keywords:
Sandwich
Vibration
Zigzag theory
Finite element
Stress continuity
LSE method

A B S T R A C T

Free and forced vibration response of laminated composite and sandwich plate having weakly bonded layer is studied by using an efficient C^0 continuous two dimensional finite element (FE) model. This FE model is based on higher order zigzag theory (HOZT) in which, in-plane displacements are obtained by superposing a global displacement field having cubical variation across thickness, on a zigzag displacement field having linear variation with a different slope in each layer. The out of plane displacement is assumed constant across the thickness of the plate. In the proposed model, transverse shear stresses are continuous at the layer interfaces and vanish at the top and bottom surfaces of the plate. At the same time, it is computationally as elegant as any single layer plate model since it requires the usual unknowns at the reference plane only. The inter-laminar imperfections are represented by in-plane displacement jumps at the layer interfaces and characterized by a linear spring layer model. In order to circumvent the problem of C^1 continuity associated with the above plate theory at the time of FE implementation, first derivatives of the transverse displacement have been treated as independent variables. It is interesting to note that the plate model having all these refined features requires unknowns at the reference plane only. The LSE method is applied to the 3D equilibrium equations of the plate problem at the post-processing stage, after in-plane stresses are calculated by using the above FE model based on HOZT for forced vibration problems. Numerical problems on free and forced vibration having different geometric and material properties of laminated composite and sandwich plates are solved. Some new problems have also been solved and new results have been presented which would be useful for future research.

1. Introduction

Structural components which are widely used in civil, mechanical, aerospace, ocean and other engineering applications are made from composites and sandwich laminates due to their advantage of high stiffness and strength to weight ratio. However, these composite (e.g., GFRP, CFRP etc.) structures are having low strength in shear due to their low shear modulus as compared to extensional rigidity. Thus in these structures, shear deformation is quite substantial. In the case of sandwich structures, it the effect becomes more complicated, since the material property variation is extremely large between the core and the face layers.

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http://dx.doi.org/10.1016/j.apm.2015.04.008

It is quite necessary to accurately interpret the structural response of laminated composite and sandwich plates for reliable and safe designs of these structures. In order to accurately represent the static and dynamic responses of these laminated composite structures, the effect of shear deformation has to be modeled very efficiently and accurately. Koiter [1] has stated the following Koiter's recommendation (KR) on 2D modeling of traditional isotropic shells on the bases of energy consideration: "a refinement of Love's first approximation theory is indeed meaningless, in general, unless the effects of transverse shear and normal strains (stresses) are taken into account at the same time." Subsequently Carrera [2,3] investigated the transverse normal strain effects on the bending and vibration of multilayered plates and shells and proposed an amendment of Koiter's recommendation (KR1): "any refinements of classical models are meaningless, in general, unless the effects of inter-laminar continuous transverse shear and normal stresses are both taken into account in a multilayered plate/shell theory."

However, most of the available refinements of classical theories [such as CLT: classical lamination theory, which is based on Kirchhoff's assumptions, and FSDT: first order shear deformation theory, which is based on the so called Reissner–Mindlin assumptions] that have been proposed for homogeneous (one-layered) and multilayered (anisotropic) plates and shells, do not account for transverse normal strain (ε_z). The main causes of violating Koiter's recommendation (KR) lie in the intrinsic coupling experienced by isotropic and orthotropic materials between in-plane stresses (σ_x, σ_y) and transverse strain (ε_z) and vice versa. Due to these couplings, it is more difficult to obtain and solve the differential equations that govern the static and dynamic behaviors of plate/shell structures compared to those theories that ignore transverse strain (ε_z). In addition to that, it is difficult to develop multilayered plate/shell theories that are able to a priori fulfill inter-laminar equilibria for both transverse shear (σ_{xz}, σ_{yz}) and normal stresses (σ_z) [4,5].

In this circumstance, a number of plate theories have been evolved to accurately model the effect of shear deformation in a refined manner. These plate theories can be sorted into two, based on their assumed displacement fields, as: (1) single layer plate theory and (2) layer-wise plate theory.

In the case of single layer plate theory ([6–13] and many more), the deformation of plate is written in terms of unknown parameters of middle plane (reference plane) of the plate. In the single layer plate theory group, the first-order shear deformation theory (FSDT) may be considered as the simplest option. Reissner and Stavasky [6] have developed a simple plate model based on FSDT which is also known as Reissner–Mindlin's plate theory. The main concern in this theory is that, it over-estimates the natural frequencies and buckling load since this theory does not look at transverse shear deformation effect, i.e. the transverse shear strain is constant across the thickness. Bert and Chen [7] have used the plate theory proposed by Reissner and Stavasky [6] and developed a close form solution to study the vibration response of simply supported anti-symmetric angle ply laminated plate. Reddy [8] was the first one who developed a FE model to study free vibration of anti-symmetric angle-ply laminated composite plates by using the Reissner–Mindlin's plate theory. Subsequently, Harik and Ekambaram [9], Bao et al. [10], Hwang and Lee [12], Goyal and Kapania [13] and many more studied the vibration and buckling behavior of composite structures based on FSDT. The FSDT assumes a constant transverse shear strain across the thickness direction and a shear correction factor is generally required to adjust the transverse shear stiffness. The accuracy of results obtained from the first order theory strongly depends on the shear correction factors [14]. These shear correction factors are sensible not only to geometric parameters of the plate, but also to the boundary and loading conditions. Shufrin and Eisenberger [15] have advised an exact solution which is based on first-order and higher order shear deformation plate theories including the effect of transverse shear deformation to study free vibration of thick rectangular plates, but accuracy of the solution is subjected to boundary conditions.

In order to preclude the use of shear correction factor and to include the actual cross-section warping of the plate, higher order shear deformation plate theory (HSDT) have been extensively developed. Kant et al. [16] derived the complete set of equations for the analysis of thick and thin elastic plates with the help of refined higher order theory. In this theory, the in-plane displacements were assumed to have a cubic variation across the plate thickness whereas the transverse displacement was assumed to vary linearly through the thickness of the plate. Later, Kant along with his co-workers extended this work for application to fibre reinforced composites and sandwich plates. However, this theory involves some additional unknowns and these unknowns do not have any physical meaning and also the number of unknowns is more than that used in FSDT. Subsequently, Reddy [17] proposed a simple higher order theory for the analysis of laminated composite plates. In this theory, in-plane displacements field were assumed to be cubic functions of thickness coordinates while the transverse displacement to be constant throughout the plate thickness. The displacement fields are chosen in such a manner so that the zero transverse shear stresses condition at top and bottom surfaces of the plate was satisfied. This theory also has the same number of independent unknowns as in the FSDT and accounts for parabolic distribution of transverse shear strains through the thickness of the plate. There is no need to use the shear correction factors in computing the shear stresses. Following the above concept of higher order shear deformation theory, Reddy and Phan [18], Putcha and Reddy [19], Tessler et al. [20], Shankara and Iyengar [21], Kant and Swaminathan [22,23], Ganapathi and Makhecha [24], Nayak et al. [25], Shi et al. [26], Khare et al. [27], Zhen and Wanji [28], Wang et al. [29], and Aydogdu [30] have investigated the problem of vibration and buckling analysis of the laminated composite and sandwich plates by using different solution techniques such as exact, analytical and finite element method. Zhen et al. [31] formulated an eight-noded C^0 finite element model to study free vibration of laminated sandwich plate. This plate theory (HSDT) consist second-order derivatives of transverse displacement (w) in the strain components, which necessitates C^1 continuity of w at the element interfaces and this would give difficulty in its finite element implementation.