Applied Mathematical Modelling 39 (2015) 5216–5229

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# Material behavior modeling with multi-output support vector regression

W. Zhao [a,*], J.K. Liu [b], Y.Y. Chen [c]

[a] Key Laboratory of Disaster Forecast and Control in Engineering, Ministry of Education of China, Jinan University, Guangzhou 510632, China
[b] Department of Mechanics, Sun Yat-sen University, Guangzhou 510275, China
[c] Earthquake Engineering Research & Test Center, Guangzhou University, Guangzhou 510405, China

ARTICLE INFO

*Article history:*
Received 10 June 2010
Received in revised form 6 March 2015
Accepted 19 March 2015
Available online 21 April 2015

*Keywords:*
Multi-support vector regression
Training
Material modeling

ABSTRACT

Based on neural network material-modeling technologies, a new paradigm, called multi-output support vector regression, is developed to model complex stress/strain behavior of materials. The constitutive information generally implicitly contained in the results of experiments, i.e., the relationships between stresses and strains, can be captured by training a support vector regression model within a unified architecture from experimental data. This model, inheriting the merits of the neural network based models, can be employed to model the behavior of modern, complex materials such as composites. Moreover, the architectures of the support vector regression built in this research can be more easily determined than that of the neural network. Therefore, the proposed constitutive models can be more conveniently applied to finite element analysis and other application fields.

As an illustration, the behaviors of concrete in the state of plane stress under monotonic biaxial loading and compressive uniaxial cycle loading are modeled with the multi-output and single-output support regression respectively. The excellent results show that the support vector regression provides another effective approach for material modeling.


## 1. Introduction

The conventional material behavior modeling methods construct mathematical models using mathematical expressions and rules to approximate the experimentally observed data, so as to capture as much as possible complex non-linear material behaviors such as ductile yielding, micro-cracking, brittle fracture, localization, strain-softening. These material models were developed almost in the same way: a mathematical model was constructed from the tested data, checked and modified against results from the other existing or new experiments. Many of them are useful to establish a general framework for understanding material behavior but weakly to capture the complex behavior of materials. Ghaboussi et al. [1–4] proposed a new method of constitutive modeling based on neural networks. This methodology was applied to the modeling of constitutive behavior of various materials [1–7], and the constructed models were employed in finite element analysis (FEA) of boundary value problems [8]. The neural network (NN) constitutive models use the learning capabilities of neural networks trained with the results of experiments, the knowledge of material behaviors are stored in the connection weights of the neural network. However, many weak points of this methodology still remain such as failure in determining of the number of processing units in the hidden layers, over fitting, and existence of many local minimal solutions. Though

* Corresponding author.

http://dx.doi.org/10.1016/j.apm.2015.03.036
0307-904X/

Ghaboussi and his coworker [5,7,9] proposed an adaptive adjustment schema of the neural network, the new model requires more training time, and the structure of the neural network cannot be ascertained in advance. To solve these problems, we present a new paradigm, called support vector regression (SVR), to provide an alternative approach of the derivation and representation of material behavior relationship.

The SVR is an application of support vector machine (SVM) in regression estimation. The SVM, a neural-network-architecture-like paradigm, developed by Vapnik and co-workers [10] in the area of statistical learning theory and structural risk minimization, has experienced a considerable sound real-world application on data classification, function approximation, regression estimation, signal processing, etc [10–12]. Based on the theory of SVM, the SVR become a well-established method for design black-box models in engineering now. The SVR has the similar architecture as the neural network (to be described later), which also has input, intermediate and output layers conform to different training rules. The maturation in research on the SVR has facilitated the development of an alternative approach to the derivation and representation of material behavior. With this new method, the knowledge of the material behavior relationship can be captured as much as possible with a combination of support vectors namely input training patterns that has been trained with the experimental data.

Commonly, the output of the realistic system or process depends on a set of factors, which are stored as a vector in the corresponding SVR model. The classical SVR considers one output at a time and the multi-output case is then dealt with by modeling each output independently of the others. This leads the relationship of the outputs that may exist between them cannot be captured in the SVR model. For this issue, Ref. [13] extended the classical SVR to multi-output systems by considering the co-kridging method. The Matérn covariance was selected as the kernel, and the multi-output case was transformed into the single-output case. In this research, we extend the classical single SVR rule to the multi-output case and apply it to training a SVR-based material model.

Considering the complexity of the material behavior, as examples of material modeling with the multi-output SVR, we have chosen to represent the biaxial and uniaxial cyclic behaviors of plain concrete just as Ref. [1] did. The units in the input and output layer of the SVR-based material model are supposed to consist of stresses, strains or even their increments, and the multi-output units are treated as a vector not a scalar, which is proved to provide a more general schema to address the problem of various material behavior modeling, especially the SVR solution of output units establish some kind of relationship due to sharing the same support vectors.

Since many uniaxial and biaxial stress–strain relations are available in the literature, to get a convenient explanation, the stress–strain data was manufactured by the formulas rather than measured experimentally. And unless otherwise indicated, all the stresses and strains refer to principal stress and principal strain.

This paper is organized as follows: Section 2 extends the classical single output SVR rule to multi-output case. Section 3 presents the SVR-based material models. Section 4 shows the examples of biaxial models of plain concrete. Section 5 gives the examples of uniaxial cyclic models for plain concrete. Section 6 provides some discussions and concluding remarks.

## 2. Support vector machine and multi-output support vector regression

Support vector machines (SVMs) are learning systems firmly grounded in the framework of statistical learning theory or VC theory, which use a hypothesis space of linear functions in a high dimensional feature space, trained with a learning algorithm from optimization theory. Within a short period of time, this learning strategy introduced by Vapnik and co-workers became competitive with the best available systems in a wide variety of real-world applications especially for pattern recognition and regression estimation. In most of these applications, SVM generalization performance (i.e. error rates on test sets) either matches or is significantly better than that of the other competing methods. The application of SVMs for density estimation and ANOVA decomposition has also been studied [10–12]. The classical SVR is the single output case, here we extend the traditional SVR rule to the multi-output case without using kridging method as [13] and give the implementation algorithm.

### 2.1. The basic idea of the multi-output support vector regression

Suppose we are given a training data set which are usually denoted by

$$(\mathbf{x}_1,\mathbf{y}_1),\ldots,(\mathbf{x}_l,\mathbf{y}_l) \in R^n \times R^m,$$

where $\mathbf{x}_i \subseteq R^n$, $\mathbf{y}_i \subseteq R^m$, $i = 1,\ldots,l$, $l$ is the number of examples. For the single output case, $m$ is an integer equal to 1 otherwise greater than 1. In $\varepsilon - SV$ regression, our goal is to find a vector composed of several functions $\mathbf{F}(\mathbf{x})$ with its components that have at most $\varepsilon$ deviation from the actually obtained targets' components of $\mathbf{y}_i$ for all the training data, and at the same time is as flat as possible.

For simplicity, we begin by describing the case of linear function $\mathbf{F}(\mathbf{x})$, taking the linear form

$$\mathbf{F}(\mathbf{x}) = \langle \mathbf{W},\mathbf{x}\rangle + \mathbf{B}, \tag{1}$$

where $\langle,\rangle$ denotes inner product, $\mathbf{W} = [\mathbf{w}_1,\ldots,\mathbf{w}_m]^T$, $\mathbf{w}_i = [w_{i1},\ldots,w_{in}]$, $i = 1,\ldots,m$, $\mathbf{B} = [b_1,\ldots,b_m]^T$. The left hand side of the formula (1) is an $m$-dimensional output function, which can also be written in its component form