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Slope stability analysis based on quantum-behaved particle swarm optimization and least squares support vector machine

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ABSTRACT

Given the complexity and uncertainty of the influencing factors of slope stability, its accurate evaluation is difficult to accomplish using conventional approaches. This paper presents the use of a least square support vector machine (LSSVM) algorithm based on quantum-behaved particle swarm optimization (QPSO) to establish the nonlinear relationship of slope stability. In the proposed QPSO-LSSVM algorithm, QPSO is employed to optimize the important parameters of LSSVM. To identify the local and global optimum, three popular benchmark functions are utilized to test the abilities of the proposed QPSO, the nonlinearly decreasing weight PSO, and the linearly decreasing weight PSO algorithms. The proposed QPSO exhibited superior performance over the other aforementioned algorithms. Simulation results obtained from PSO-LSSVM, QPSO-LSSVM, and LSSVM algorithms are compared in a case. Case analysis shows that QPSO-LSSVM has the quickest search velocity and the best convergence performance among the three algorithms, and is therefore considered most suitable for slope stability analysis.

1. Introduction

Slope stability analysis is an essential task in the design and construction of water conservancy and civil engineering structures. Slope stability is characterized by complex failure mechanisms and nonlinear dynamic performance because of the influence of different factors, such as geomorphic characteristics, geologic properties, and engineering. Limit equilibrium [1,2] and finite element [3,4] have been widely used for determining slope stability to reduce or prevent damages caused by landslides. Slope stability is difficult to evaluate accurately using conventional analysis approaches because the influencing factors of slope stability are complex and uncertain. Artificial neural network (ANN) [5,6] and support vector machine (SVM) [7–9] have been successfully used in recent years to solve slope stability problems. Limitations such as overfitting, slow convergence speed, and poor generalization ability seriously hamper the practical application of ANN [10]. By contrast, SVM, which is based on statistical learning theory, is characterized by having non-linear kernels, high generalization ability, and sparse solution [11]. Overcoming the shortcomings of ANN, SVM improves computational efficiency and precision [12]. To reduce the complexity of the optimization process, a modified version, i.e., a least squares support vector machine (LSSVM), is proposed by taking constraints with equality rather than inequality to obtain a linear set of equations rather than a quadratic programming (QP) problem in the dual space [13]. LSSVM presents similar advantages as SVM, but an additional advantage of LSSVM is that it requires solving a set of only linear equations (linear programming), which is much

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easier and computationally more simple than quadratic programming. Thus, LSSVM has excellent potential application in slope stability analysis.

LSSVM has been used for slope stability analysis without considering how important parameters of LSSVM are determined [14]. Several parameters must be obtained for the LSSVM to achieve a high level of performance. Motivated by this problem, many researchers have drawn ideas from the field of biology. A number of such biology-inspired evolutionary techniques have been developed, such as genetic algorithm (GA) [15–18] and particle swarm optimization (PSO) [19,20], which are widely used for solving optimization problems. PSO is originally attributed to Kennedy and Eberhart, who were inspired by the behavior of bird swarms in 1995 [21]. Both GA and PSO have been used extensively for various optimization problems [22–25]. PSO outperforms GA in multivariable function optimization because complex operations such as selection, crossing and mutation are not required in PSO [26–30]. Since 1995, many attempts have been made to improve the performance of the PSO [31–37]. Sun et al. [38,39] introduced quantum theory into PSO and proposed a quantum-behaved PSO (QPSO) algorithm, which is a global search algorithm that, in theory, is guaranteed capable of finding good optimal solutions in the search space. Compared with PSO, the iterative equation of QPSO needs no velocity vectors for particles, has fewer parameters to adjust, and can be implemented more easily. The results of experiment on widely used benchmark functions indicate that the QPSO is a promising algorithm [38,39] that exhibits better performance than the standard PSO.

Extensive literature reviews suggest that the proposed hybridization of QPSO and LSSVM into slop stability analysis is a novel approach. In the present paper, QPSO is employed to optimize the parameters of LSSVM, and a hybrid QPSO-LSSVM algorithm that combines QPSO and LSSVM is proposed for slope stability analysis.

The rest of this paper is structured as follows. Sections 2 and 3 describe PSO and QPSO, respectively. Section 4 discusses the regression algorithm of LSSVM and an improved grid search algorithm. Section 5 presents the PSO-LSSVM and QPSO-LSSVM algorithms for slope stability analysis. Section 6 demonstrates a case to measure the performance of the proposed QPSO algorithm. Section 7 compares the QPSO-LSSVM algorithm with PSO-LSSVM and LSSVM algorithms with the use of training and testing samples of slope stability analysis. Lastly, Section 8 provides the conclusions drawn in this paper.

2. Particle swarm optimization

PSO is a heuristic global optimization algorithm broadly applied in optimization problems. PSO is developed on a very simple theoretical framework that is easily implemented with only primitive mathematical operators [32]. In PSO, a group of particles is composed of m particles in D dimension space where the position of the particle i is $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$ and the speed is $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$. The speed and position of each particle are changed in accordance with the following equation:

$$v_{id}^{j+1} = wv_{id}^{j} + c_1 r_1 \left(p_{id}^{j} - x_{id}^{j}\right) + c_2 r_2 \left(p_{gd}^{j} - x_{id}^{j}\right), \tag{1}$$

$$x_{id}^{j+1} = x_{id}^{j} + v_{id}^{j+1}, \tag{2}$$

where $i = 1, 2, \ldots, m;\ d = 1, 2, \ldots, D$; m is the particle size; p_{id}^{j} is the dth dimension component of the *pbest* that is the individual optimal location of the particle i in the jth iteration; p_{gd}^{j} is the dth dimension component of the *gbest* that is the optimal position of all particles in the jth iteration; w is the inertia weight coefficient; c_1 and c_2 are learning factors; r_1 and r_2 are random numbers in the range [0, 1].

The inertia weight w, which balances the global and local exploitation abilities of the swarm, is critical for the performance of PSO. A large inertia weight facilitates exploration but slows down particle convergence. Conversely, a small inertia weight facilitates fast particle convergence it sometimes leads to the local optimal. The most popular algorithm for controlling inertia weight is linearly decreasing inertia weight PSO [37]. The strategy of linearly decreasing inertia weight is widely used to improve the performance of PSO, but this approach has a number of drawbacks [33]. Several adaptive algorithms for tuning inertia weight have been presented [33–36]. In the present work, we propose the method of nonlinearly decreasing inertia weight to tune the value of w for further performance improvement as follows:

$$w = w_{\max} - (w_{\max} - w_{\min}) \times (t-1)^2/(t_{\max} - 1)^2, \tag{3}$$

where $w_{\max}$ and $w_{\min}$ are the maximum and minimum values of w, respectively; t is the current iteration number; and $t_{\max}$ is the maximum iteration number.

3. Quantum-behaved particle swarm optimization

The main disadvantage of the PSO algorithm is that global convergence is not guaranteed [40]. To address this problem, Sun et al., inspired by the trajectory analysis of PSO and quantum mechanics, developed and proposed the QPSO algorithm [38,39]. Particles move according to the following iterative equation:

$$x_{i,j}(t+1) = p_{i,j}(t) + \alpha \cdot \left|mbest_j(t) - x_{i,j}(t)\right| \cdot \ln(1/u) \quad \text{if} \quad k \geq 0.5, \tag{4}$$

$$x_{i,j}(t+1) = p_{i,j}(t) - \alpha \cdot \left|mbest_j(t) - x_{i,j}(t)\right| \cdot \ln(1/u) \quad \text{if} \quad k < 0.5, \tag{5}$$