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Analysis of the validity of the three-point off-axis bending method

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ABSTRACT

Several methods for determining in-plane shear properties of anisotropic materials (i.e. fibre reinforced composite materials and natural wood) based on flexural loading have been proposed. One of those methods is the off-axis flexural test that considers unidirectional composite materials subject to three-point bending loading. As a result of the anisotropic behaviour, unidirectional off-axis laminates subjected to flexure present a bending–twisting coupling that may cause, in the case of 3-point bending loading, a lift-off of the specimen on the fixture supports. Such specimen lift-off has been considered one of the critical features to be considered on the mentioned experimental method. Besides geometrical parameters, material elastic constants, as longitudinal, transversal and shear elastic moduli, influence the validity to use this method for characterising in-plane shear properties. On that sense, an analytical approach for studying the material conditions for which the application of the off-axis 3P-B test is adequate is presented.

1. Introduction

One of the most important parameters to be considered for studying anisotropic materials (i.e. fibre reinforced composite materials, natural wood and wood products, and mammal bones, among others) are the in-plane shear properties: in-plane shear modulus and in-plane shear strength. On that sense, several test methods are currently available for measuring in-plane (or intralaminar) shear properties of anisotropic materials. Those methods consider a variety of specimen geometries, applied loading, and material configurations. The seven more accepted methods for determining these in-plane shear properties are:

- The $[\pm 45^\circ]_{2s}$ tensile test: proposed by Petit [1], simplified by Rosen [2], and described in ASTM D3518 standard [3].
- The 10° off-axis tensile test: proposed by IIT Research Institute [4], presented by Chamis and Sinclair [5], and analysed by Pagano and Halpin [6].
- The two-rail and three-rail shear tests: proposed by Whitney et al. [7], and described in ASTM D3518 standard [3].
- The thin-walled tube torsion test: studied by Whitney et al. [8,9], and described in ASTM D5448 standard [10]. Moreover, Tsai et al. [11] proposed a torsion test in a thin rectangular cross-section specimen.

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Nomenclature

Greek letters

γ_{mid}	condition related to the material for the validity of the 3-point off-axis bending method
Λ	longitudinal to transversal elastic modulus ratio
T	transversal elastic modulus to shear modulus ratio
θ	fibre orientation angle

Upper cases

E	elastic modulus
G	shear modulus
K'	experimental failure point
L	span of the specimen
L'	length of the specimen
P	applied load
P_f	experimental failure load
$R_{ij.yy}$	displacement field influence ratio on the related to the displecement due to S_{yy} in the middle point, where $i,j = x,y,s$
S'_{ij}	out-of-plane compliance coefficients related to transverse shear, where $i,j = 4,5$
S_{ij}	in-plane compliance coefficient, where $i,j = x,y,s$, and x,y are oriented some angle respect to fibre direction
A, B, C, D	corner points and their respective specimen parts divided according to symmetry planes

Lower cases

b	width of the specimen
c	span to width ratio
c_{LO}	critical span to width ratio for lift-off to occur
c_{mid}	critical span-to-width ratio for the point load condition
g	span to overall length ratio
h	height of the specimen
k	ratio between twisting moments when lift-off does not occur and when it does, $k = 1$ if lift-off occurs
m	experimental slope of the load–displacement curve relative to the load application point
w_M	displacement field due to bending and twisting
w_Q	displacement field caused by transverse shear forces
x,y,z	coordinates along length, width and thickness, respectively
x_0,y_0	normalised coordinates
$y_{0K'}$	relative width coordinate of the experimental location of the failure point

Indexes

$ij.yy$	displacement field influence ratio on the related to the displecement due to S_{yy} in the middle point, where $i,j = x,y,s$
L	longitudinal
LO	lift-off at the support
M	displacement field due to bending and twisting
mid	condition to verify applied point load
$Mrel$	displacement field (due to bending and twisting) relative to the middle point
Q	displacement field caused by transverse shear forces
$Qrel$	displacement caused by transverse shear forces relative to the middle point
s	shear
T	transversal

- The Iosipescu test: initially proposed to study isotropic metallic materials [12], was adopted by Walrath and Adams [13] for testing 3-D carbon–carbon composite materials. This method is described in ASTM D5379 standard [14].
- The Arcan test: it is similar to the Iosipescu test [15].
- The V-notched rail shear test: proposed by Adams et al. [16], and described in ASTM D7078 standard [17].

All these tests have been surveyed, described in detail, and compared in terms of advantages and drawbacks, which make them non-ideal methods but complementary ones.

In recent years, other methods for determining the in-plane shear properties under flexural loading have been proposed, being the most important: