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Probability and convex set hybrid reliability analysis based on active learning Kriging model

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ABSTRACT

Probability and convex set hybrid reliability analysis (HRA) is investigated in this paper. It is figured out that a surrogate model only rightly predicting the sign of the performance function can meet the demand of HRA in accuracy. According to this idea, a methodology based on active learning Kriging model called ALK-HRA is proposed. When constructing the Kriging model, the proposed method only approximates the performance function in some region of interest, i.e., the region where the sign of response tends to be wrongly predicted. Then Monte Carlo Simulation (MCS) is performed based on the Kriging model. ALK-HRA is very accurate for HRA with calling the performance function as few times as possible. Three numerical examples are investigated to demonstrate the efficiency and accuracy of the presented method, which include two simple problems and one complicated engineering application.

1. Introduction

Traditional probabilistic reliability analysis (PRA) requires precise probability distributions of the uncertain parameters, which may be impossible for some parameters because of limited experimental samples. In that case, unjustified assumptions in constructing a probabilistic model may yield misleading results in PRA [1,2]. With limited experimental samples, the convex model was elaborated by Ben-Haim and Elishakoff [3,4]. In this model, the uncertain parameters are described by interval or ellipsoid model. For interval model, the fluctuation of variables is assumed to fall in the space between their lower and upper bounds. As for ellipsoid model, the variables are assumed to vary in a hyper-ellipsoid. For convenience, the variables described by the convex model are uniformly referred to as convex variables in this paper. Based on the theory of convex model, the so-called non-probabilistic reliability approaches have been deeply investigated [5–10] and become efficient supplements to the traditional PRA [11,12].

In practical engineering, a frequently encountered case is that [11–16]: some of the uncertainties can be characterized with certain probabilistic model and others have to be treated by convex model. In such circumstance, the probabilistic and non-probabilistic hybrid reliability synthesis is needed.

To tackle the reliability problem with both random and interval variables, many approaches have been proposed. Qiu et al. [14] and Wang et al. [15] respectively advanced a probabilistic and interval hybrid reliability model through interval arithmetic. Du [16] suggested a double-loop procedure with fully nested interval analysis and probabilistic analysis. To reduce the computational cost, Du [16,17] decoupled the double-loop procedure into a sequential single-loop optimization

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procedure. Sensitivity analysis for hybrid reliability with both random and interval variables was carried out in Refs. [13,18]. Recently, a new analysis method [19] and an enhanced unified analysis approach [20] were proposed based on the first order reliability method (FORM).

As the literature survey shows, the existing researches mainly focus on the hybrid reliability with both random variables and interval variables. However, interval model is only the simplest specific instance of convex model and it does not account for the dependence among the uncertainties [9–12]. To tackle the combination of random variables with the general case of convex variables, Luo et al. [11] proposed a probability and convex set mixed reliability model. In their model, some of the convex variables are described as interval variables and some are represented by ellipsoid model considering their dependence. The minimum reliability index was advanced to measure the reliability of a structure in their model. The mathematical programming method (MPM) and the single-loop iterative (SLI) method were proposed to compute the minimum reliability index. The minimum reliability index was used as the constraint in reliability-based design optimization (RBDO) with both random variables and convex variables in the authors' later works [12,21].

The minimum reliability index was defined as the distance from the origin to the minimum limit state surface in the standard normal space of random variables. Just as reliability index behaves in PRA, the minimum reliability index cannot accurately measure the reliability of a structure with highly nonlinear performance functions in HRA. In fact, only the maximum failure probability corresponding to the minimum limit state surface can accurately measure the reliability of a structure in HRA. However, the existing numerical algorithms are all oriented to obtaining the minimum reliability index. For performance functions which are highly nonlinear or have multiple design points, these algorithms will be very inaccurate. Moreover, these algorithms are all gradient-based optimization algorithms and the efficiency decreases as the number of uncertain variables increases.

This paper aims to develop a more efficient and accurate method for HRA with both random and convex variables. Firstly, it is figured out that a surrogate model only rightly predicting the sign of the performance function can meet the demand of HRA in accuracy. Then an active learning Kriging model for HRA (ALK-HRA) is elaborated. When constructing the Kriging model, we do not approximate the performance function throughout the uncertain space, but only in some region of interest, i.e., the region where the sign of response tends to be wrongly predicted. Then Monte Carlo Simulation (MCS) can be effectively implemented based on the Kriging model.

The remainder of this paper is organized as follows. The preliminary of HRA with both random and convex variables is presented in Section 2. HRA with MCS method is proposed in Section 3 which will be the benchmark of the proposed method. Section 4 is devoted to the proposed ALK-HRA methodology. Three examples are investigated to demonstrate the accuracy and efficiency of the presented method in Section 5. Conclusions are made in the last section.

2. Probability and convex set hybrid reliability analysis

2.1. Non-probabilistic convex model

Interval and ellipsoid are two types of the most widely used convex models to describe the convex variables. When the variables are independent with each other, interval model is employed. When some extent of dependence exists among several variables, the ellipsoid model is used. In a general case that some variables are independent with each other while some variables are correlated, the so-called multi-ellipsoid convex model is competent for the description.

In the theory of multi-ellipsoid convex model, it is needed to classify the variables into groups according to their dependence. In the same group the variables are correlated while the variables are uncorrelated in different groups. Denote the vector of convex variables as $\boldsymbol{Y} = [y_1, y_2, \ldots, y_{N_Y}]^T$. Suppose that the variables are classified into N_E groups, then $\boldsymbol{Y}$ is expressed as

$$\boldsymbol{Y}^T = \left[\boldsymbol{Y}_1^T, \boldsymbol{Y}_2^T, \ldots, \boldsymbol{Y}_{N_E}^T\right], \tag{1}$$

where $\boldsymbol{Y}_i$ denotes the vector of variables located in the ith group. If n_i variables exist in the ith group, there is $\sum_{i=1}^{N_E} n_i = N_Y$. In the ith group, the uncertain domain of the variables is quantified with an ellipsoid as [9]

$$\Omega_i = \left\{\boldsymbol{Y}_i | (\boldsymbol{Y}_i - \bar{\boldsymbol{Y}}_i)^T \boldsymbol{M}_i (\boldsymbol{Y}_i - \bar{\boldsymbol{Y}}_i) \leqslant 1\right\}, \tag{2}$$

where $\bar{\boldsymbol{Y}}_i$ is the nominal value of $\boldsymbol{Y}_i$, $\boldsymbol{M}_i$ is an $n_i \times n_i$ symmetric positive-definite matrix called the characteristic matrix of an ellipsoid. The uncertain domain of all the convex variables is

$$\Omega = \{\boldsymbol{Y} | \boldsymbol{Y}_i \in \Omega_i \ (i = 1, 2, \ldots, N_E)\}. \tag{3}$$

On how to create a reasonable ellipsoid through limited experimental date of uncertain variables, the readers can refer to Ref. [9].

Obviously, if a group consists of only one variable which means the variable is uncorrelated with others, the characteristic matrix of the group degenerates into a scalar and the ellipsoid model degenerates into an interval model. Hence, the multi-ellipsoid convex model provides a uniform representation accommodating both dependent and independent variables.