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Scheduling jobs with position and sum-of-processing-time based processing times

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ABSTRACT

The paper is devoted to some single-machine scheduling problems with variable job processing times. The objectives are to minimize the makespan (i.e., the maximum completion time of all jobs), and to minimize the total completion time. For some special cases, we show that these problems can be solved in polynomial time. For some another special cases of the makespan and the total completion time minimization problems, we prove that an optimal schedule has an V-shape property in terms of processing times. We also propose a heuristic algorithm by utilizing the V-shape property.

1. Introduction

Scheduling problems and models with learning effect (learning curve) and/or deteriorating jobs (time-dependent processing times) have been paid more attention in recent years [1–5]. Extensive surveys of scheduling models and problems involving learning effect (learning curve) and/or deteriorating jobs (time-dependent processing times) are provided by Alidaee and Womer [2], Cheng et al. [3], Gawiejnowicz [4], and Biskup [5]. More recent papers which have considered scheduling problems and models with learning effect (learning curve) and/or deteriorating jobs (time-dependent processing times) include Wang and Cheng [6], Eren and Guner [7,8], Eren [9], Wang et al. [10,11], Yang and Chand [12], Janiak and Rudek [13], Wu and Lee [14], Lee and Wu [15], Wang et al. [16], Wang and Guo [17], Wang et al. [18–20], Wang and Wang [21], Shen et al. [22], Jiang et al. [23], Lu and Wang [24], Wang and Wang [25], Wang et al. [26], Ji et al. [27], Lu et al. [28], Wang and Wang [29,30], Wang and Liu [31], Yin et al. [32], Huang and Wang [33], Wang and Wang [34], Wang et al. [35], Yin et al. [36,37], Wang and Wang [38], and Bai et al. [39].

"*In general, jobs scheduled in different positions should be subject to different effects of human learning. In the early stage of processing a given set of jobs, the worker is not familiar with the operations, so the learning effect on the jobs scheduled early is not apparent. On the other hand, when the worker has spent more time processing jobs, his learning improves. So the worker's learning effect on a job depends not only on the total processing time of the jobs that he has processed but also on the job's position*" [28]. In this study we focus on the scheduling problems considered by Wu and Lee [14], but with different parameters. We show that

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some special cases of minimization of the total completion time and minimization of the makespan are polynomially solvable. For some another special cases, we show that an optimal schedule has an V-shape property in terms of processing times. Excluding the trivial V-shape properties in problems containing a common due-date for all jobs, optimal V-shape properties are quit rare. Hence, any new schedule with this property has some theoretical and practical significance. In some cases it reduces significantly the number of candidates for an optimal schedule, and enables the development of efficient and accurate heuristics (see Gordon et al. [40,41]).

The paper is organized as follows. In Section 2, we formulate the model. In Section 3, we consider several single-machine scheduling problems. In Section 4, we propose a heuristic algorithm utilized the V-shape property for the total completion time minimization problem, and followed by a computational experiment. The last section is the conclusion.

2. Problem statement

We aim at scheduling n independent and non-preemptive jobs $J = \{J_1, J_2, \ldots, J_n\}$ on a single machine. It is assumed that all the jobs are available at time zero and without any overlapping and idle time between them. As in Wu and Lee [14], we assume that the actual processing time for job J_j is given by

$$p_{j[r]} = p_j\left(1 + \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2}, \tag{1}$$

where p_j is the normal processing time of job J_j $(j = 1, 2, \ldots, n)$, $p_{[r]}$ is the normal processing time of a job if scheduled in the rth position in a sequence, a_1 and a_2 are given constant number respectively, $a_1 \geqslant 0\,(a_2 \geqslant 0)$ in the case of deterioration and $a_1 \leqslant 0\,(a_2 \leqslant 0)$ in the case of learning, and $\sum_{i=1}^{0} p_{[i]} := 0$. Under a schedule $\pi = [J_1, J_2, \ldots, J_n]$, let $C_j = C_j(\pi)$ be the completion time of job J_j. The objective is to find a schedule π^* such that $C_{\max}(\pi^*) \leqslant C_{\max}(\pi^*)$ and $\sum C_j(\pi^*) = \sum_{l=1}^{n} C_l(\pi^*)$ for any schedule π respectively, where $C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}$ and $\sum C_j = \sum_{l=1}^{n} C_l$ represent makespan and total completion time, respectively. Using the traditional three-field notation introduced by Graham et al. [42], the problems under study are denoted as $1|p_{j[r]} = p_j\left(1 + \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2}|C_{\max}$ and $1|p_{j[r]} = p_j\left(1 + \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2}|\sum_{l=1}^{n} C_l$, respectively.

3. The main results

Lemma 1.

(i) $\lambda - 1 + \delta(1 + \lambda x)^a - \delta\lambda(1 + x)^a \geqslant 0$ *if* $a \leqslant 0,\ \lambda \geqslant 1,\ 0 \leqslant \delta \leqslant 1$ *and* $x \geqslant 0$.
(ii) $\lambda - 1 + \delta(1 + \lambda x)^a - \delta\lambda(1 + x)^a \geqslant 0$ *if* $a \geqslant 1,\ \lambda \geqslant 1,\ 0 \leqslant \delta \leqslant 1$ *and* $x \geqslant 0$.
(iii) $\lambda - 1 + \delta(1 + \lambda x)^a - \delta\lambda(1 + x)^a \leqslant 0$ *if* $0 \leqslant a \leqslant 1,\ \lambda \geqslant 1,\ \delta \geqslant 1$ *and* $x \geqslant 0$.

Proof. See the proof of Lemma 2 in Wu and Lee [14]. □

3.1. Case i: $0 \leqslant a_1 \leqslant 1$ and $a_2 \geqslant 0$

Theorem 1. *For the* $1|p_{j[r]} = p_j\left(1 + \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2},\ 0 \leqslant a_1 \leqslant 1,\ a_2 \geqslant 0|C_{\max}$ *problem, an optimal schedule can be obtained by sequencing the jobs in non-increasing order of* p_j *(i.e., the largest processing time (LPT) first rule in* $O(n \log n)$ *time).*

Proof. The proof are using an idea that after switching two adjacent jobs not in the LPT order, we can improve the objective function.

Let $\pi = [S_1, J_j, J_k, S_2]$ and $\pi' = [S_1, J_k, J_j, S_2]$, where S_1 and S_2 are partial sequences, and $p_j \geqslant p_k$. We also assume that there are $r - 1$ jobs in S_1, i.e., for schedule π, J_j and J_k are the rth and the $(r + 1)$th jobs, respectively. For schedules π and π', let A denote the completion time of the last job in S_1, then the completion times of J_j and J_k are

$$C_j(\pi) = A + p_j\left(1 + \frac{\sum_{i=1}^{r-1} p_{[i]}}{\sum_{i=1}^{n} p_i}\right)^{a_1} r^{a_2},$$

$$C_k(\pi) = A + p_j\left(1 + \frac{\sum_{i=1}^{r-1} p_{[i]}}{\sum_{i=1}^{n} p_i}\right)^{a_1} r^{a_2} + p_k\left(1 + \frac{\sum_{i=1}^{r-1} p_{[i]} + p_j}{\sum_{i=1}^{n} p_i}\right)^{a_1} (r + 1)^{a_2}, \tag{2}$$