Applied Mathematical Modelling 39 (2015) 4022–4032

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Horizontal well's path planning: An optimal switching control approach

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ARTICLE INFO

Article history:
Received 7 May 2013
Received in revised form 10 June 2014
Accepted 10 December 2014
Available online 20 December 2014

Keywords:
Horizontal well's path planning
Optimal switching control
Continuous state inequality constraint
Time-scaling transformation
Computational method

ABSTRACT

In this paper, we consider a three-dimensional horizontal well's path planning problem, where the well's path evolves as a combination of several constant-curvature smooth turn segments. The problem is formulated as an optimal switching control problem subject to continuous state inequality constraints. By applying the time-scaling transformation and constraint transcription in conjunction with local smooth approximation technique, the optimal switching control problem is approximated by a sequence of optimal parameter selection problems with only box constraints, each of which is solvable by gradient-based optimization techniques. The optimal path planning problems of the wells Ci-16-Cp146 and Jin27 in Liaohe oil field are solved to demonstrate the applicability of the approach proposed.

1. Introduction

In practice, a well path is a three-dimensional (3-D) curve, rather than lying on a plane, where the well path is described as a combination of turn sections and straight sections. The goal of planning a horizontal well's path is to construct a trajectory that reaches a given target at a specified inclination and azimuth from a given starting location, subject to various constraints arising from engineering specifications. Several well planning softwares are available commercially for finding a horizontal well's path. However, most of them use a trial-and-error procedure to obtain a solution. In addition to being time-consuming and depending on user-experience, these techniques are limited to simple well's paths, and may not generate an optimal path, as the trial-and-error search is usually user-driven.

Mathematical optimization theory provides a much more sophisticated, rigorous, and efficient approach to the well's path planning [1]. An optimal control approach is proposed to determine a least-length trajectory in [2]. A sequential unconstrained minimization technique is presented to determine a minimum-length path for a two-dimensional S-shaped well, subject to build and drop-rate restrictions [3]. A procedure using nonlinear optimization theory is developed for 3-D well paths and path corrections in [4]. In [5–8], it is assumed that the 3-D well's path is expressed as a combination of several constant-curvature smooth turn segments. A nonlinear multistage dynamical system is thus proposed to describe the 3-D well's path. Taking the weighted sum of the target error and the length of the well's path as the cost function, a multistage optimal control problem subject to continuous state inequality constraints is investigated. For this multistage optimal control problem, the optimality conditions are derived via nonsmooth optimization theory in [9] and maximum principle in

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http://dx.doi.org/10.1016/j.apm.2014.12.014
0307-904X/

[10]. However, the solution methods used in [5–10] are based on either heuristic or direction search methods. They are computationally expensive with poor convergence properties. In addition, the continuous state inequality constraints arising from engineering specifications are ignored in [9,10]. Consequently, some so-called optimal solutions obtained actually fail to satisfy the continuous state inequality constraints at some points along the well's path. This is clearly undesirable in practice.

In this paper, we formulate the 3-D horizontal well's path planning problem as an optimal switching control problem subject to continuous state inequality constraints. We develop a new solution method. First, by the time-scaling transformation [11], the optimal switching control problem is transformed equivalently into an optimal control problem with fixed switching instants. Then the constraint transcription is used in conjunction with local smoothing approximation technique [12] to approximate the constrained optimal control problem as a sequence of optimal parameter selection problems with only box constraints. Each of these optimal parameter selection problems is solvable by gradient-based optimization methods. For illustration, two practical optimal path planning problems of the wells Ci-16-Cp146 and Jin27 in Liaohe oil field in China are solved by using the proposed approach. Our results show that for each case, the length of the whole path is reduced by around 3%–4% and the precision of reaching the target is higher compared with the best results obtained previously in the literature. Moreover, the continuous state inequality constraints arising from the engineering specifications are fulfilled everywhere.

2. Problem formulation

Consider a 3-D horizontal well's path planning as shown in Fig. 1. The well's path, which is required to reach a given target from the kick-off point (i.e., the location of the point where the curve begins to deviate from vertical) at a specified inclination and azimuth, is described in the Cartesian coordinate system, with $\bar{x}$-axis representing North/South (positive $\bar{x}$ being North), $\bar{y}$-axis representing East/West (positive $\bar{y}$ being East), and the $\bar{z}$-axis ($\bar{z}$ positive downwards) representing the true vertical depth (TVD). The arc length from the kick-off point is denoted by s, and any point $P(s)$ on the curve is described by its inclination $\alpha(s)$ and azimuth $\varphi(s)$, and coordinate $(\bar{x}(s), \bar{y}(s), \bar{z}(s))$. The tool-face angle ω and the curvature K are the decision parameters. In line with practical situation, we assume that.

(H_1) The well's path is a combination of n smooth turn segments.
(H_2) The curvature K_i and the tool-face angle ω_i are constants in each ith turn segment, $i = 1, \ldots, n$.
(H_3) The design of the well trajectory is only for non-straight horizontal well. That is to say, the inclination $\alpha(s) \notin (k\pi - \varepsilon_0, k\pi + \varepsilon_0)$, $k = 0, 1, \ldots$, where $\varepsilon_0 > 0$ is a given real number. Therefore, we may assume, without loss of generality, that $\alpha(s) = k\pi + \varepsilon_0$ whenever $|\alpha(s) - k\pi| \leqslant \varepsilon_0, k = 0, 1, \ldots$.

Under Assumptions (H_1)-(H_3), the change rates of $\alpha(s)$ and $\varphi(s)$ obey, respectively, the following rules [6]:

$$\dot{\alpha}(s) = K\cos\omega, \quad \dot{\varphi}(s) = \frac{K\sin\omega}{\sin\alpha(s)}. \tag{1}$$

Furthermore, the change rates of $\bar{x}(s), \bar{y}(s)$ and $\bar{z}(s)$ with respect to arc length s are given as follows [6]:

$$\dot{\bar{x}}(s) = \sin\alpha(s)\cos\varphi(s), \quad \dot{\bar{y}}(s) = \sin\alpha(s)\sin\varphi(s), \quad \dot{\bar{z}}(s) = \cos\alpha(s). \tag{2}$$

Now, let $x(s) = (\alpha(s), \varphi(s), \bar{x}(s), \bar{y}(s), \bar{z}(s))^\top$, $\xi^i = (K_i, \omega_i), i = 1, \ldots, n$, and

$$f(x(s), \xi^i) = \begin{pmatrix} \xi_1^i \cos\xi_2^i \\ \frac{\xi_1^i \sin\xi_2^i}{\sin x_1(s)} \\ \sin x_1(s)\cos x_2(s) \\ \sin x_1(s)\sin x_2(s) \\ \cos x_1(s) \end{pmatrix}. \tag{3}$$

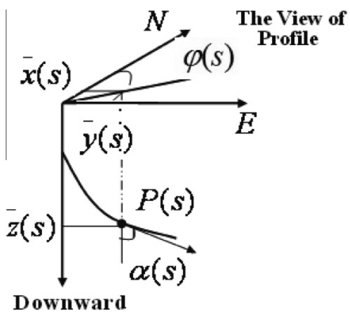

Fig. 1. Coordinate system for well's path.