A mathematical model of meat cooking based on polymer–solvent analogy

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A R T I C L E   I N F O

A B S T R A C T

Mathematical modelling of transport phenomena in food processes is vital to understand the process dynamics. In this work, we study the process of double sided cooking of meat by developing a mathematical model for the simultaneous heat and mass transfer. The constitutive equations for the heat and mass transport are based on Fourier conduction, and the Flory–Huggins theory respectively, formulated for a two-phase transport inside a porous medium. We investigate a reduced one-dimensional case to verify the model, by applying appropriate boundary conditions. The results of the simulation agree well with experimental findings reported in literature. Finally, we comment upon the sensitivity of the model to the porosity of meat.

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1. Introduction

Since antiquity, thermal processing has remained the technology of choice to improve the eating quality and safety of food products, and to extend their shelf-life. Although within recent years research in food science has largely focused on development of nonthermal technologies [1], thermal processing still remains the most widely used method in the food industries [2]. Of special importance in thermal processing of foods are the meat and meat products. This is because these almost invariably undergo thermal processing at some stage before consumption (with some exceptions). In addition, meat is eaten on a daily basis in many countries. The composition and high moisture content of meat creates an ideal environment for the growth and proliferation of pathogenic and spoilage microorganisms [3]. The major objective of thermal processing and cooking is to guarantee food safety by killing bacteria (for example *Escherichia coli* O157:H7 and *Listeria monocytogenes*) and inactivating their enzymes or other metabolites in foods. The U.S. Department of Agriculture Food Safety and Inspection Service (USDA-FSIS) comparative risk assessment of nonintact and intact beef steaks indicated that oven broiling to an internal temperature of more than 60 °C would result in safe blade-tenderized beef steaks [4,5]. A common household method for cooking meat (example beef, hamburger or patties) involves simultaneous application of heat from both sides (double sided cooking).

The purpose of a mathematical model of the heat and mass transfer in a food production process is to describe the physical processes as accurately as possible for the given food production process ([6]). In order to be able to optimise the cooking of meat, in general, and a beef steak in particular (which is at focus in this study), it is important to develop a well-posed representative mathematical model. In recent past, the importance of mathematical modelling in cooking/roasting process

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of meat and meat products has been well-emphasised by many researchers [7–11]. The mathematical models encountered in food science literature often are either empirical or mechanistic (physics-based) [12]. Empirical models are generally obtained from simple mathematical correlation of experimental data, and therefore are data driven. On the other hand, the elementary processes of heat and mass transfer are considered in mechanistic models. The advantage of physics-based mechanistic models is that these provide insight into the physical processes in a manner that is more precise [13].

Many foods, in general, are described as porous matrices by virtue of their structural/cellular arrangement [14,15]. Meat is composed of bundles of muscle fibres, which are largely protein. The intercellular space within muscle tissues is primarily filled with blood plasma which renders a pressure in the pores when driven by external forces. The plasma can be considered as a newtonian liquid. On the other hand, within the intracellular region the water is bound to the muscle proteins. The water distribution and porous structure of meat can be appreciated from the details provided by [16,11,17]. Based on the above discussion, it does not come as a surprise that the moisture movement within meat could be dealt with a continuum approach despite the porous structure. Flow in a porous medium is classically described by Darcy's equation for the liquid velocity; nevertheless, appropriate problem-specific couplings and amendments to the equation are also common (for example [18]). The flow under this scenario is driven by a pressure gradient. Extending the porous description of muscle, and drawing analogies from soft condensed matter physics, [19,8] deduced a model for heat and mass transport during cooking of meat. This approach considers that meat is composed of a polymer matrix made of protein and that the Flory–Rehner theory holds true. Under these assumptions, the moisture driving entity when cooking would be the swelling pressure which can be substituted into Darcy's equation. This approach provides far more accurate predictions than the lumped Fickian diffusion considered hitherto [20] and also agrees well with the capillary pressure driven flow description for moisture transport in meat [21]. Inspired by van der Sman's pioneering works, in conjunction with experiments conducted by [22], we focus upon the development of a two-phase, soft condensed matter physics based mechanistic model to simulate the transport phenomena within a beef steak when simultaneously heated from both sides. The presented work differs from [19,8] in several ways. While van der Sman studied the convective oven cooking of meat using the Flory–Rehner theory, the present work aims at numerically simulating the double sided conductive pan cooking of beef using the simplified Flory–Huggins theory. In addition, we formulate the conservation equations for a two phase mixture (as opposed to the Darcy's porous medium transport alone in [19,8]) of a polymer and a solvent, both of which are dealt with a continuum approach. Our approach in dealing the protein and solvent phases separately via a two phase model allowed assessing the effect of porosity of meat on cooking time, which has not been studied hitherto. This dynamics of porous structure is of particular importance when heating smaller pieces of meat, as identified earlier [8]. Furthermore, the model now has sufficient physics to capture the details of the underlying processes, giving better results for thin steaks, even with a one dimensional geometry. We wish to point that the model derived herein has some similarities to, and is based on the polymer–solvent models derived elsewhere in the literature in the context of biofilms; see for example, [23,24].

The structure of the present paper is the following. In the next section, we derive the model based on conservation laws, introduce the polymer–solvent assumption based on Flory–Huggins theory and define the boundary conditions. We then non-dimensionalise, and proceed further by reducing the model to a simpler form. We then study the reduced model in one dimension, and compare with experimental results reported by [22]. Finally, we evaluate the sensitivity of the model to model parameters and discuss the consequences.

2. Model derivation

In Fig. 1, a pictorial presentation of the problem in consideration and the geometry is provided for the case where meat is subjected to double sided heating. The lean beef sample (<4% fat) is heated from top and bottom using hot plates maintained at a constant temperature. It is assumed that the surface of meat is completely in contact with the hot plate. Under this scenario, the heat flows from either end towards the centre ($0 < z < L$). As mentioned earlier, the pores are ideally filled with plasma and ions and other soluble proteins. For the present study, we assume that the pores are saturated with water (incompressible fluid) and the solid matrix does not undergo global deformation. We now define ρ_s to be the density of the protein matrix, whose volume fraction is ϕ. Satisfying the criteria of saturation, we have $(1 - \phi)$, the volume fraction
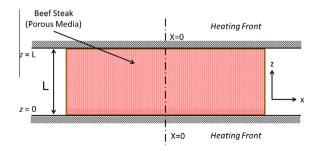

Fig. 1. Geometry of the problem for double sided heating.