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Derivation of soliton solutions to nonlinear evolution equations using He's variational principle

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ABSTRACT

In this study, He's semi-inverse variational principle is applied to obtain traveling wave solutions of the coupled nonlinear Schrödinger type equations. Both solitary and periodic wave solutions are observed by a suitable choice of the parameters involved. This method provides a powerful tool for nonlinear wave equations.

1. Introduction

Many important phenomena and dynamic processes in physics, mechanics, chemistry, and biology can be described by nonlinear partial differential equations (NLPDEs) such as Korteweg Vries equations, Burgers equations, Schrödinger equations, and Boussinesq equations. Thus, it would be useful to obtain a mathematical algorithm for determining the exact solutions of NLPDEs. Many effective methods can be used to construct traveling wave solutions of NLPDEs, such as the Adomian decomposition method [1], homotopy perturbation method [2], variational iteration method [3,4], He's variational approach [5], extended homoclinic test approach [6,7], homogeneous balance method [8–11], Jacobi elliptic function method [12–15], Băclund transformation [16,17], and G'/G expansion method [18]. In the present study, we use He's semi-inverse variational principle to obtain traveling wave solutions for coupled nonlinear Schrödinger type equation that contain solitary and periodic wave solutions based on a suitable choice of parameters.

It is important to note that a new constrained variational principle for heat conduction was obtained recently based on the semi-inverse method combined with the separation of variables [19], which is exactly the same as He-Lee's variational principle [20]. A short remark on the history of the semi-inverse method in the establishment of a generalized variational principle is given in [21].

The variational approach is used to search for solitary solutions of nonlinear differential equations, nonlinear differential–difference equations, and nonlinear fractional differentials, thereby providing physical insights into the nature of the problem solution. He's variational principle is a very effective and convenient method for identifying the necessary variational principles for nonlinear physical problems based directly on field equations. Using the variational formula obtained, we can easily obtain solitons by the Ritz method [22].

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The remainder of this paper is organized as follows. In Section 2, we give a brief algorithm for He's semi-inverse variational principle. In Section 3, we apply this method to coupled nonlinear Schrödinger type equations and obtain several exact solutions for this model. We give our conclusion in Section 4.

2. Algorithm for He's semi-inverse method

For a given nonlinear equation

$$H(u, u_t, u_x, u_{xt}, \ldots) = 0. \quad (1)$$

The main steps of this method are as follows.

Step 1. Using the wave transformation $u(x,t) = U(\zeta),\ \zeta = x - ct + \zeta_0$, we can convert Eq. (1) into an ordinary differential equation:

$$H(U, U_\zeta, U_{\zeta\zeta}, \ldots) = 0. \quad (2)$$

Step 2. If possible, we integrate Eq. (2) one or more times, thereby yielding the constant(s) of integration. For simplicity, we set the integration constant(s) to zero.

Step 3. According to He's semi-inverse method, we construct the following:

$$J = \int_0^\infty L d\zeta, \quad (3)$$

where L is a Lagrangian for Eq. (2).

Step 4. By the Ritz method [22], we can obtain different forms of the solitary wave solutions, such as $U(\zeta) = A sech(B\zeta),\ U(\zeta) = A csch(B\zeta)$, and $U(\zeta) = A\tanh(B\zeta)$. In this study, we obtain a solitary wave solution in the form of:

$$U(\zeta) = A\, sech(B\zeta), \quad (4)$$

where A and B are constants that need to be determined.

By substituting Eq. (4) into Eq. (3) and making J stationary with respect to A and B yields

$$\frac{\partial J}{\partial A} = 0, \quad (5)$$

$$\frac{\partial J}{\partial B} = 0. \quad (6)$$

By solving Eqs. (5) and (6), we obtain A and B. Hence, the solitary wave solution Eq. (4) is determined.

3. Application to coupled nonlinear Schrödinger type equations

Now, we consider the new coupled nonlinear Schrödinger type (CNLST) equations:

$$\begin{aligned} u_{xt} &= u_{xx} + \frac{2}{1-\beta^2}|u|^2 u + u(v - w), \\ v_t &= -\frac{(|u|^2)_t}{1+\beta} + (1+\beta)v_x, \\ w_t &= \frac{(|u|^2)_t}{1-\beta} + (1-\beta)w_x, \end{aligned} \quad (7)$$

where β is a real constant, $|\beta| \neq 1$, which was proposed by Ma and Geng based on a spectral problem and its auxiliary problem [23].

Using the transformation

$$u(x,t) = U(\zeta)e^{i(k_2 x - \omega_2 t + \varphi_0)}, \quad v(x,t) = V(\zeta), \quad w(x,t) = W(\zeta), \quad (8)$$

where $\zeta = x - ct + \xi_0,\ \varphi_0, \xi_0$ are constants.

By substituting Eq. (8) into Eq. (7), we obtain

$$(1-c)U'' - k_2(k_2 + \omega_2)U + \frac{2}{1-\beta^2}U^3 + U(V - W) = 0, \quad (9)$$

$$-cV' - \frac{2c}{1+\beta}UU'' + (1+\beta)V' = 0, \quad (10)$$

$$-cW' + \frac{2c}{1-\beta}UU' + (1-\beta)W' = 0, \quad (11)$$