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Hygrothermal vibration of orthotropic double-layered graphene sheets embedded in an elastic medium using the two-variable plate theory

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ABSTRACT

Vibration of orthotropic double-layered graphene sheets under hygrothermal conditions is investigated in this paper using the trigonometric shear deformation plate theory. This theory accounts for sinusoidal distribution of transverse shear stress, and satisfies the free transverse shear stress conditions on the top and bottom surfaces of the plate without using shear correction factor. Unlike the conventional shear deformation theory, the present trigonometric theory contains only two unknowns. The two layers are assumed to be bonded by an internal elastic medium and surrounded by external elastic foundations. The equations of motion are derived based on the nonlocal theory. The motion equations contain the small scale effect as will as hygrothermal effect. The present solution is examined by comparing the present results with those being in the open literature. The effects played by small scale parameter, temperature rise, the degree of moisture concentration, plate aspect ratio and side-to-thickness ratio are studied.

1. Introduction

Graphene is an actually two-dimensional atomic crystal with exceptional electronic and mechanical properties. Many nanostructures based on the carbon such as carbon nanotube, nanorings, nanoplates, nanobeams, etc. are considered as deformed graphene sheet. So, analysis of graphene sheets is a basic matter in the study of the nanomaterials. To design plate efficiently, understanding their vibration behavior is extremely important. Vibration of scale-free plates has been studied widely in the literatures using local theories. But these theories cannot predict the size effects on the nanostructures with small size. Thus, the nonlocal elasticity theory introduced by Eringen [1] is developed. He modified the classical continuum mechanics for taking into account small scale effects. In this theory, the stress state at a given point depends on the strain states at all points, while in the local theory, the stress state at any given point depends only on the strain state at that point. The nonlocal elasticity theory has been widely applied to study the various behavior of nanostructures. The continuum plate model is used by Kitipornchai et al. [2] to investigate the mechanical analysis of graphene sheets. He et al. [3] demonstrated vibration analysis of multi-layered graphene sheets (MLGSs) in which the van der Waals interaction between layers is described by an explicit formula. The nanoscale vibrational analysis of MLGSs embedded in an elastic medium based on the classical plate theory has been studied by Behfar and Naghdabadi [4]. Lu et al. [5] employed the nonlocal Kirchhoff

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and Mindlin plate theories to derive the basic equations of simply supported nanoplates. Duan and Wang [6] investigated the axisymmetric bending of circular nanoplates using a nonlocal plate theory. Aydogdu [7] investigated free vibration of simply supported multi-walled carbon nanotubes using the generalized shear deformation theory. Pradhan and Phadikar [8] illustrated vibration analysis of MLGSs embedded in polymer matrix employing nonlocal continuum mechanics. Murmu and Pradhan [9] investigated in-plane vibration of nanoplates employing nonlocal continuum mechanics and considering small scale effect. Based on the nonlocal plate model, the vibrational characteristics of MLGSs with different boundary conditions embedded in an elastic medium were investigated by Ansari et al. [10] using finite element method. Aksencer and Aydogdu [11] proposed Levy type solution for vibration and buckling of nanoplate with isotropic property using the classical plate theory. Further, Sobhy [12] introduced Levy type solution to study the bending response of SLGSs in thermal environment using the two-variable plate theory. Wang et al. [13] studied the thermal effect on vibration of double-layered nanoplate with isotropic mechanical properties. Thermal vibration analysis of orthotropic nanoplates based on nonlocal continuum mechanics was studied by Satish et al. [14] using two variable refined plate theory. Based on a generalized form of Kirchhoff plate model, an analytical method has been presented by Assadi [15] to study the size dependent forced vibration of rectangular nanoplates under general external loading. The elastic buckling and vibration characteristics of isotropic and orthotropic nanoplates have been investigated by Analooei et al. [16] employing finite strip method. Based on the sinusoidal shear deformation plate theory (SPT), Zenkour and Sobhy [17] studied the thermal buckling of SLGSs embedded in an elastic medium. Thermomechanical bending and free vibration of single-layered graphene sheets (SLGSs) embedded in an elastic medium were studied by Sobhy [18] employing the SPT with five unknowns. While, in Sobhy [19], the free vibration, mechanical buckling and thermal buckling responses of MLGSs were investigated using new two-variable plate theories with two unknowns. In this context, Yan et al. [20] utilized a higher-order gradient theory to investigate the free vibration characteristics of single-wall carbon nanocones.

Structures, in general, are more often exposed to high temperature as well as moisture during manufacture and use. The varying environmental conditions due to moisture absorption and temperature seem to have an adverse effect on the stiffness and strength of the structural composites. The rise in moisture and temperature reduces the elastic moduli of the material and induces internal initial stresses, which may affect the stability as well as the safety of the structures [21]. Hence, the changes in deformations due to the hygrothermal effect seem to be an important consideration in composite analysis and design. Based on classical plate theory, Whitney and Ashton [22] studied the effect of hygrothermal conditions on the bending, buckling and vibration of simply supported composite laminated plates using the Ritz method. Based on the first-order shear deformation theory, Sai Ram and Sinha [23] studied the hygrothermal effects on the free vibration behavior of laminated composite plates employing finite element method. Lee et al. [24] employed the classical plate theory to investigate the hygrothermal effects on the cylindrical bending of symmetric angle-ply laminated plates subjected to uniform transverse load for different boundary conditions. Chang [25] developed a linear hygrothermoelastic theory to analyze transient responses in an infinitely long solid cylinder subjected to hygrothermal loadings. Further, Chang and Weng [26] analyzed the transient response of an axisymmetric double-layer annular cylinder subjected to hygrothermal loading. Hygrothermal effect on dynamic interlaminar stresses in laminated plates with piezoelectric actuators has been investigated by Wang et al. [27]. Bahrami and Nosier [28] investigated the interlaminar hygrothermal stresses in laminated plates. Brischetto [29] studied the hygrothermoelastic analysis of multilayered composite and sandwich shells. Zenkour [30] studied the hygrothermal bending analysis for a functionally graded material plate. Zenkour et al. [31] studied the effect of hygrothermal conditions on the antisymmetric cross-ply laminates using a unified shear deformation plate theory. Mashat and Zenkour [32] investigated the hygrothermal bending response of the sector-shaped annular plate with variable radial thickness based on Kirchhoff plate theory. The closed-form exact solution for the hygrothermal response of inhomogeneous piezoelectric hollow cylinders subjected to both a mechanical load and an electric potential has been obtained by Zenkour [33].

A limited number of papers concerns the study of the effect of moisture conditions on nanostructures. Woo et al. [34] investigated experimentally the residual properties of epoxy-nanoclay nanocomposites exposed to UV light and moisture. It is found from this study that the flexural modulus decreases due to the effect of moisture. The effects of temperature and moisture on the dynamic failure responses of laminated graphite epoxy composites subjected to high strain rate loading were investigated by Wosu et al. [35]. Yao et al. [36] illustrated the effect of water adsorption on the electrical properties of graphene oxide films using experimental measurements. They found that, at low humidity, graphene exhibit poor conductivity, whereas at high humidity, the conductivity of graphene increases. Further, on the basis of experimental study, Han et al. [37] introduced the moisture-responsive graphene paper using the self-controlled photoreduction. This study shows that the graphene sheet exhibits moisture-responsive properties due to selective adsorption of water molecules. The hygrothermo-mechanical bending of single-layered graphene sheets embedded in an elastic medium has been investigated by Alzahrani et al. [38] employing the sinusoidal shear deformation plate theory.

There are many shear deformation plate theories in open literature (see, e.g., [39–47]) developed to study the various behavior of plates. Although these theories give sufficiently accurate results, they contain at least five unknown functions leading to five equations of motion that may be complicated. Thus, there is a scope to introduce a shear deformation theory that is simple to use. A new shear deformation plate theory using only two unknown functions has been established by Shimpi [48]. Unlike any other theory, this theory contains two unknowns and so only two governing equations are obtained. Moreover, it is similar to the classical plate theory (CPT) in many aspects as governing equation, boundary conditions and moment expressions. Nevertheless, it gives results, for various values of side-to-thickness ratio, not only more accurate than