



# Hopf bifurcation and stability for predator–prey systems with Beddington–DeAngelis type functional response and stage structure for prey incorporating refuge



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## ABSTRACT

A kind of stage-structured predator–prey model with Beddington–DeAngelis functional response incorporating a prey refuge is investigated in this paper. By analyzing the corresponding characteristic equations, the local stability of the equilibria is investigated. Moreover, Hopf bifurcations occur at the positive equilibrium as the delay  $\tau$  crosses some critical values. Further, the influence of prey refuge on densities of predator species and prey species is investigated. Numerical simulations are carried out to illustrate our main results.

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## 1. Introduction

Predator–prey models are very important models in population dynamics, they have been studied by both mathematicians and ecologists. In the natural world, species always experience the immature stage and mature stage of their life, this case performs different kinds of characteristics at each stage of growth. Therefore, the ecological models with stage-structure are more reasonable than the ones without stage-structure, in other words, delay differential equations exhibit much more complicated dynamical behaviors than ordinary differential equations. The existed literature [1] had shown that a time delay could destroy the stability of the positive equilibrium and caused the fluctuation of population, thereby caused Hopf bifurcation. It is obvious that taking the effective factors of stage structure and time delay into account is more meaningful when we study the interaction of population. The models with stage-structure and time delay have been considered by many authors (see [2–6] and the references therein). Especially, Ref. [2] discussed a predator–prey model of Lotka–Volterra type with time delays and stage structure for prey

$$\begin{aligned}\dot{x}_i(t) &= -dx_i(t) + rx_m(t), \\ \dot{x}_m(t) &= -d_2x_m(t) + bx_i(t) - c_1x_m^2(t) - a_1x_m(t)y(t - \tau_1), \\ \dot{y}(t) &= -d_3y(t) + a_2x_m(t - \tau_2)y(t) - c_2y^2(t).\end{aligned}\tag{1}$$

By analyzing the corresponding characteristic equations, the local stability of the equilibrium for model (1) is investigated, and Hopf bifurcations occurring at the positive equilibrium as the delay  $\tau$  crosses some critical values are demonstrated.

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Furthermore, by using Kamke comparison theorem and an iteration technique, some conditions for the global asymptotical stability of the equilibrium are derived in [2].

From the view of the interaction of population, taking the real environment factor into account, density constraints inside the same species, the functional response of predator on prey, these influence factors should be a function of saturation item of predator to prey. Therefore, the mixed models with stage-structure, time delays and functional response are much more consistent with the real world, and they have been paid much attention by many scholars widely (see [7–11]). Ref. [7] investigated a stage-structured predator–prey model with Holling type-III functional response and time delay due to the gestation of the predator

$$\begin{aligned} \dot{x}_1(t) &= ax_2(t) - r_1x_1(t) - bx_1(t), \\ \dot{x}_2(t) &= bx_1(t) - r_2x_2(t) - b_1x_2^2(t) - \frac{a_1x_2^2(t)y(t)}{1 + mx_2^2(t)}, \\ \dot{y}(t) &= \frac{a_2x_2^2(t - \tau)y(t - \tau)}{1 + mx_2^2(t - \tau)} - ry(t). \end{aligned} \tag{2}$$

By using of the characteristic equations, the local stability of each of feasible equilibrium of model (2) is discussed, and the existence of a Hopf bifurcation at the coexistence equilibrium is established. Formulas are derived to determine the direction of Hopf bifurcations and the stability of the bifurcating periodic solutions by means of the normal form theory and center manifold theorem.

In many cases, predators have to search for food to share or compete for food, in this case, the functional response in predator–prey models should be predator-dependent, and the Beddington–DeAngelis type functional response has performed even better with the following model in [12] (also see [13])

$$\begin{aligned} \dot{x}_1(t) &= rx_2(t) - d_1x_1(t) - ax_1^2(t) - bx_1(t), \\ \dot{x}_2(t) &= bx_1(t) - d_2x_2(t) - a_{11}x_2^2(t) - \frac{a_{12}x_2(t)x_3(t)}{1 + mx_2(t) + nx_3(t)}, \\ \dot{y}(t) &= -d_3x_3(t) + \frac{a_{21}x_2(t)x_3(t)}{1 + mx_2(t) + nx_3(t)} - a_{22}x_3^2(t). \end{aligned} \tag{3}$$

Motivated by the work [2,7,12], a kind of stage-structure predator–prey models with Beddington–DeAngelis type functional response and time delay incorporating prey refuge

$$\begin{aligned} \dot{x}_1(t) &= ax_2(t) - rx_1(t) - bx_1(t), \\ \dot{x}_2(t) &= bx_1(t) - cx_2(t) - \alpha x_2^2(t) - \frac{\beta(1 - m)x_2(t)y(t)}{a_1 + b_1(1 - m)x_2(t) + c_1y(t)}, \\ \dot{y}(t) &= \frac{d\beta(1 - m)x_2(t - \tau)y(t - \tau)}{a_1 + b_1(1 - m)x_2(t - \tau) + c_1y(t - \tau)} - \gamma y(t), \end{aligned} \tag{4}$$

is investigated in our paper, where  $x_1(t)$ ,  $x_2(t)$  and  $y(t)$  denote the densities of immature prey species, mature prey species and predator species at time  $t$  respectively;  $a$  is the intrinsic growth rate of immature prey species;  $r$ ,  $c$  and  $\gamma$  denote the death rates of immature prey species, mature prey species and predator species respectively;  $b$  is the transformation rate from immature prey species to mature prey species;  $\alpha$  is intra-species competition rate of mature prey species;  $\beta(1 - m)$  is the capturing rate of the mature predator;  $d$  is the conversion rate of nutrients into the production of predator species;  $\tau$  is the time of predator species for pregnancy;  $m \in [0, 1]$  is refuge rate to prey, and  $(1 - m)x_2(t)$  leaves the prey species available to the predator species; the predator species consumes the prey species with Beddington–DeAngelis functional response incorporating prey refuge  $\frac{\beta(1 - m)x_2(t)y(t)}{a_1 + b_1(1 - m)x_2(t) + c_1y(t)}$ , and  $\frac{d\beta(1 - m)x_2(t - \tau)y(t - \tau)}{a_1 + b_1(1 - m)x_2(t - \tau) + c_1y(t - \tau)}$  denotes the growth rate of predator species which are pregnant at time  $t - \tau$ .

The initial conditions for model (4) take the form

$$(x_1(\theta), x_2(\theta), y(\theta)) = (\varphi_1(\theta), \varphi_2(\theta), \psi(\theta)) \in C_+ = ([-\tau, 0], R_+^3), \varphi_1(0), \varphi_2(0), \psi(0) > 0, \tag{5}$$

where  $R_+^3 = \{(x_1, x_2, y) \in R_+^3 : x_1 \geq 0, x_2 \geq 0, y \geq 0\}$ .

Our main purpose is to reveal the dynamical behaviors of model (4) in this paper, such as Hopf bifurcation, periodic solutions and global stability and so on. Our paper is organized as follows: the local stability of the equilibrium and the existence of Hopf bifurcation at positive equilibrium are concerned in Section 2; the influence of refuge rate on the densities to predator species and prey species is discussed in Section 3; numerical simulations are illustrated to show our main results in Section 4.

## 2. Local stability and Hopf bifurcation

Throughout this paper, we always assume the following conditions are valid.

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